

# (Non)renormalizability of the D-deformed Wess-Zumino model

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## Abstract

We continue the analysis of the  $D$ -deformed Wess-Zumino model which was started in the previous paper. The model is defined by a deformation which is non-hermitian and given in terms of the covariant derivatives  $D_\alpha$ . We calculate one-loop divergences in the two-point, three-point and four-point Green functions. We find that the divergences in the four-point function cannot be absorbed and thus our model is not renormalizable. We discuss possibilities to render the model renormalizable.

**Keywords:** supersymmetry, non-hermitian twist, deformed Wess-Zumino model, supergraph technique, renormalization

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# 1 Introduction

Having in mind problems which physics encounters at small scales (high energies), in recent years attempts were made to combine supersymmetry (SUSY) with noncommutative geometry. Different models were constructed, see for example [1], [2], [3], [4], [5], [6], [7], [8], [9], [10]. Some of these models emerge naturally as low energy limits of string theories in a background with a constant Neveu-Schwarz two form and/or a constant Ramond-Ramond two form. For some references on noncommutative geometry and non(anti)commutative field theories see [7].

One way to describe a noncommutative deformation is to consider the algebra of functions on a smooth manifold with the usual pointwise multiplication replaced by a noncommutative  $\star$ -product. A wide class of  $\star$ -products can be defined by twists  $\mathcal{F}$ . The notion of twist was first introduced in [11] in the context of quantum groups; recently it has been used to describe symmetries of noncommutative spaces, see for example [6], [12], [13].

In our previous paper [7] we started the analysis of a simple model. Since we are interested in renormalizability properties of the supersymmetric theories with twisted symmetry, we introduce the non(anti)commutative deformation via the twist

$$\mathcal{F} = e^{\frac{1}{2}C^{\alpha\beta}D_\alpha \otimes D_\beta}. \quad (1.1)$$

Here  $C^{\alpha\beta} = C^{\beta\alpha} \in \mathbb{C}$  is a complex constant matrix and  $D_\alpha = \partial_\alpha + i\sigma_{\alpha\dot{\alpha}}^m \bar{\theta}^{\dot{\alpha}} \partial_m$  are the SUSY covariant derivatives. The twist (1.1) is not hermitian under the usual complex conjugation. Due to our choice of the twist, the coproduct of the SUSY transformations remains undeformed, leading to the undeformed Leibniz rule. The inverse of (1.1) defines the  $\star$ -product. It is obvious that the  $\star$ -product of two chiral fields will not be a chiral field. Therefore we have to use projectors to separate chiral and antichiral parts. The deformed Wess-Zumino action is then constructed by inclusion of all possible invariants under the deformed SUSY transformations.

The plan of the paper is as follows. In the next section we summarize the most important properties of our model, the details of the construction are given in [7]. Using the background field method and the supergraph technique in Sections 3 and 4 we calculate the one-loop divergences in two-point and three-point functions. We obtain that, on the level of one-point and two-point functions, all divergences can be absorbed in the counterterms. There is no mass renormalization and no tadpole diagrams appear. These results are in agreement with the results obtained in [7]. In order to cancel divergences in the three-point function, a new SUSY invariant interaction term is added to the classical action. Proceeding to the four-point function in Section 5, we see that divergences cannot be cancelled even with the new term. In the final section, we give some comments and compare our results with results already present in the literature. Some details of our calculations are presented in appendix.

## 2 $D$ -deformed Wess-Zumino model

We work in the superspace generated by  $x^m$ ,  $\theta^\alpha$  and  $\bar{\theta}_{\dot{\alpha}}$  coordinates which fulfill

$$[x^m, x^n] = [x^m, \theta^\alpha] = [x^m, \bar{\theta}_{\dot{\alpha}}] = 0, \quad \{\theta^\alpha, \theta^\beta\} = \{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\} = \{\theta^\alpha, \bar{\theta}_{\dot{\alpha}}\} = 0, \quad (2.2)$$

with  $m = 0, \dots, 3$  and  $\alpha, \beta = 1, 2$ . These coordinates we call supercoordinates, to  $x^m$  we refer as bosonic and to  $\theta^\alpha$  and  $\bar{\theta}_{\dot{\alpha}}$  we refer as fermionic coordinates. We work in Minkowski space-time with the metric  $(-, +, +, +)$  and  $x^2 = x^m x_m = -(x^0)^2 + (x^1)^2 + (x^2)^2 + (x^3)^2$ .

A general superfield  $F(x, \theta, \bar{\theta})$  can be expanded in powers of  $\theta$  and  $\bar{\theta}$ ,

$$\begin{aligned} F(x, \theta, \bar{\theta}) = & f(x) + \theta \phi(x) + \bar{\theta} \bar{\chi}(x) + \theta \theta m(x) + \bar{\theta} \bar{\theta} n(x) + \theta \sigma^m \bar{\theta} v_m(x) \\ & + \theta \theta \bar{\theta} \bar{\lambda}(x) + \bar{\theta} \bar{\theta} \theta \varphi(x) + \theta \theta \bar{\theta} \bar{\theta} d(x). \end{aligned} \quad (2.3)$$

Under the infinitesimal SUSY transformations it transforms as

$$\delta_\xi F = (\xi Q + \bar{\xi} \bar{Q}) F, \quad (2.4)$$

where  $\xi^\alpha$  and  $\bar{\xi}_{\dot{\alpha}}$  are constant anticommuting parameters and  $Q^\alpha$  and  $\bar{Q}_{\dot{\alpha}}$  are SUSY generators,

$$Q_\alpha = \partial_\alpha - i \sigma_{\alpha\dot{\alpha}}^m \bar{\theta}^{\dot{\alpha}} \partial_m, \quad \bar{Q}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} + i \theta^\alpha \sigma_{\alpha\dot{\alpha}}^m \partial_m. \quad (2.5)$$

As in [8], [13] we introduce a deformation of the Hopf algebra of infinitesimal SUSY transformations by choosing the twist  $\mathcal{F}$  in the following way

$$\mathcal{F} = e^{\frac{1}{2} C^{\alpha\beta} D_\alpha \otimes D_\beta}, \quad (2.6)$$

with the complex constant matrix  $C^{\alpha\beta} = C^{\beta\alpha} \in \mathbb{C}$ . Note that this twist<sup>1</sup> is not hermitian,  $\mathcal{F}^* \neq \mathcal{F}$ ; the usual complex conjugation is denoted by " $*$ ". It can be shown that (2.6) satisfies all requirements for a twist, [14]. The Hopf algebra of infinitesimal SUSY transformations does not change since

$$\{Q_\alpha, D_\beta\} = \{\bar{Q}_{\dot{\alpha}}, D_\beta\} = 0. \quad (2.7)$$

This means that the full supersymmetry is preserved.

The inverse of the twist (2.6),

$$\mathcal{F}^{-1} = e^{-\frac{1}{2} C^{\alpha\beta} D_\alpha \otimes D_\beta}, \quad (2.8)$$

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<sup>1</sup>Strictly speaking, the twist  $\mathcal{F}$  (2.6) does not belong to the universal enveloping algebra of the Lie algebra of infinitesimal SUSY transformations. Therefore to be mathematically correct we should enlarge the algebra by introducing the relations for the operators  $D_\alpha$  as well. In this way the deformed SUSY Hopf algebra remains the same as the undeformed one. However, since  $[D_\alpha, M_{mn}] \neq 0$  the super Poincaré algebra becomes deformed and different from the super Poincaré algebra in the commutative case.

defines the  $\star$ -product. For arbitrary superfields  $F$  and  $G$  the  $\star$ -product reads

$$\begin{aligned}
F \star G &= \mu_\star \{F \otimes G\} \\
&= \mu \{ \mathcal{F}^{-1} F \otimes G \} \\
&= F \cdot G - \frac{1}{2} (-1)^{|F|} C^{\alpha\beta} (D_\alpha F) \cdot (D_\beta G) \\
&\quad - \frac{1}{8} C^{\alpha\beta} C^{\gamma\delta} (D_\alpha D_\gamma F) \cdot (D_\beta D_\delta G),
\end{aligned} \tag{2.9}$$

where  $|F| = 1$  if  $F$  is odd (fermionic) and  $|F| = 0$  if  $F$  is even (bosonic). The second line is in fact the definition of the multiplication  $\mu_\star$ . No higher powers of  $C^{\alpha\beta}$  appear since derivatives  $D_\alpha$  are Grassmanian. The  $\star$ -product (2.9) is associative<sup>2</sup>, noncommutative and in the zeroth order in the deformation parameter  $C_{\alpha\beta}$  it reduces to the usual pointwise multiplication. One should also note that it is not hermitian,

$$(F \star G)^* \neq G^* \star F^*. \tag{2.11}$$

The  $\star$ -product (2.9) leads to

$$\begin{aligned}
\{\theta^\alpha \star \theta^\beta\} &= C^{\alpha\beta}, \quad \{\bar{\theta}_\alpha \star \bar{\theta}_\beta\} = \{\theta^\alpha \star \bar{\theta}_\alpha\} = 0, \\
[x^m \star x^n] &= -C^{\alpha\beta} (\sigma^{mn} \varepsilon)_{\alpha\beta} \bar{\theta} \bar{\theta}, \\
[x^m \star \theta^\alpha] &= -i C^{\alpha\beta} \sigma_{\beta\dot{\beta}}^m \bar{\theta}^{\dot{\beta}}, \quad [x^m \star \bar{\theta}_\alpha] = 0.
\end{aligned} \tag{2.12}$$

The deformed superspace is generated by the usual bosonic and fermionic coordinates (2.2) while the deformation is contained in the new product (2.9). From (2.12) it follows that both fermionic and bosonic part of the superspace are deformed. This is different from [8] where only fermionic part of the superspace was deformed.

The deformed infinitesimal SUSY transformation is defined in the following way

$$\delta_\xi^\star F = (\xi Q + \bar{\xi} \bar{Q}) F. \tag{2.13}$$

Since the coproduct is not deformed, the usual Leibniz rule follows. The  $\star$ -product of two superfields is again a superfield; its transformation law is given by

$$\begin{aligned}
\delta_\xi^\star (F \star G) &= (\xi Q + \bar{\xi} \bar{Q}) (F \star G) \\
&= (\delta_\xi^\star F) \star G + F \star (\delta_\xi^\star G).
\end{aligned} \tag{2.14}$$

Being interested in a deformation of the Wess-Zumino model, we need to analyze properties of the  $\star$ -products of chiral fields. A chiral field  $\Phi$  fulfills  $\bar{D}_{\dot{\alpha}} \Phi = 0$ , where

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<sup>2</sup>The associativity of the  $\star$ -product follows from the cocycle condition [14] which the twist  $\mathcal{F}$  has to fulfill

$$\mathcal{F}_{12}(\Delta \otimes id)\mathcal{F} = \mathcal{F}_{23}(id \otimes \Delta)\mathcal{F}, \tag{2.10}$$

where  $\mathcal{F}_{12} = \mathcal{F} \otimes 1$  and  $\mathcal{F}_{23} = 1 \otimes \mathcal{F}$ . It can be shown that the twist (2.6) indeed fulfills this condition, see [15] for details.

$\bar{D}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} - i\theta^{\alpha}\sigma_{\alpha\dot{\alpha}}^m\partial_m$  and  $\bar{D}_{\dot{\alpha}}$  is related to  $D_{\alpha}$  by the usual complex conjugation. In terms of the component fields,  $\Phi$  is given by

$$\begin{aligned}\Phi(x, \theta, \bar{\theta}) &= A(x) + \sqrt{2}\theta^{\alpha}\psi_{\alpha}(x) + \theta\theta H(x) + i\theta\sigma^l\bar{\theta}(\partial_l A(x)) \\ &\quad - \frac{i}{\sqrt{2}}\theta\theta(\partial_m\psi^{\alpha}(x))\sigma_{\alpha\dot{\alpha}}^m\bar{\theta}^{\dot{\alpha}} + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}(\square A(x)).\end{aligned}\quad (2.15)$$

The  $\star$ -product of two chiral fields reads

$$\begin{aligned}\Phi \star \Phi &= \Phi \cdot \Phi - \frac{1}{8}C^{\alpha\beta}C^{\gamma\delta}D_{\alpha}D_{\gamma}\Phi D_{\beta}D_{\delta}\Phi \\ &= \Phi \cdot \Phi - \frac{1}{32}C^2(D^2\Phi)(D^2\Phi) \\ &= A^2 - \frac{C^2}{2}H^2 + 2\sqrt{2}A\theta^{\alpha}\psi_{\alpha} \\ &\quad - i\sqrt{2}C^2H\bar{\theta}_{\dot{\alpha}}\bar{\sigma}^{m\dot{\alpha}\alpha}(\partial_m\psi_{\alpha}) + \theta\theta(2AH - \psi\psi) \\ &\quad + C^2\bar{\theta}\bar{\theta}\left(-H\square A + \frac{1}{2}(\partial_m\psi)\sigma^m\bar{\sigma}^l(\partial_l\psi)\right) \\ &\quad + i\theta\sigma^m\bar{\theta}\left(\partial_m(A^2) + C^2H\partial_m H\right) \\ &\quad + i\sqrt{2}\theta\theta\bar{\theta}_{\dot{\alpha}}\bar{\sigma}^{m\dot{\alpha}\alpha}(\partial_m(\psi_{\alpha}A)) \\ &\quad + \frac{\sqrt{2}}{2}\bar{\theta}\bar{\theta}C^2(-H\theta\square\psi + \theta\sigma^m\bar{\sigma}^n\partial_n\psi\partial_m H) \\ &\quad + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}(\square A^2 - \frac{1}{2}C^2\square H^2),\end{aligned}\quad (2.16)$$

where  $C^2 = C^{\alpha\beta}C^{\gamma\delta}\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta}$ . Due to the  $\bar{\theta}$ ,  $\bar{\theta}\bar{\theta}$  and the  $\theta\theta\bar{\theta}$  terms in (2.16),  $\Phi \star \Phi$  is not a chiral field. Following the method developed in [8] we decompose all  $\star$ -products of the chiral fields into their irreducible components by using the projectors defined in [16].

Finally, in order to write a SUSY invariant action we collect all invariant terms. Thus we propose the following action

$$\begin{aligned}S &= \int d^4x \left\{ \Phi^+ \star \Phi \Big|_{\theta\theta\bar{\theta}\bar{\theta}} + \left[ \frac{m}{2} \left( P_2(\Phi \star \Phi) \Big|_{\theta\theta} + 2a_1 P_1(\Phi \star \Phi) \Big|_{\bar{\theta}\bar{\theta}} \right) \right. \right. \\ &\quad + \frac{\lambda}{3} \left( P_2(P_2(\Phi \star \Phi) \star \Phi) \Big|_{\theta\theta} + 3a_2 P_1(P_2(\Phi \star \Phi) \star \Phi) \Big|_{\bar{\theta}\bar{\theta}} \right. \\ &\quad \left. \left. + 2a_3 (P_1(\Phi \star \Phi) \star \Phi) \Big|_{\theta\theta\bar{\theta}\bar{\theta}} + a_4 P_1(\Phi \star \Phi) \star \Phi^+ \Big|_{\bar{\theta}\bar{\theta}} \right) + \text{c.c.} \right\}.\end{aligned}\quad (2.17)$$

Coefficients  $a_1, \dots, a_4$  are real and constant. Note that (2.17) is the full action, i.e. no higher order terms in the deformation parameter  $C^{\alpha\beta}$  appear. Compared with the action which we constructed in the previous paper [7], action (2.17) has one additional interaction term, the term  $P_1(\Phi \star \Phi) \star \Phi^+ \Big|_{\bar{\theta}\bar{\theta}}$  which is allowed by the invariance under the SUSY transformations (2.13) and goes to zero in the commutative limit. Its presence is not required by renormalizability of the two-point function and that is why it was

not considered in [7]. However, as we shall see, in order to have a renormalizable three-point function we have to add one additional term to the classical action and this term is precisely  $P_1(\Phi \star \Phi) \star \Phi^+ \Big|_{\bar{\theta}\bar{\theta}}$ .

### 3 The one-loop effective action and the supergraph technique

In this section we look at the quantum properties of our model. To be more precise, we calculate the one-loop divergent part of the effective action up to second order in the deformation parameter. We use the background field method, dimensional regularization and the supergraph technique. Note that the use of the supergraph technique significantly simplifies calculations.

In order to apply the supergraph technique, the classical action (2.17) has to be rewritten as an integral over the superspace. The kinetic part takes the form

$$S_0 = \int d^8z \left\{ \Phi^+ \Phi + \left[ -\frac{m}{8} \Phi \frac{D^2}{\square} \Phi + \frac{ma_1 C^2}{8} (D^2 \Phi) \Phi + c.c. \right] \right\}, \quad (3.18)$$

while the interaction is given by

$$\begin{aligned} S_{int} = \lambda \int d^8z \left\{ -\Phi^2 \frac{D^2}{12\square} \Phi + \frac{a_2 C^2}{8} \Phi \Phi (D^2 \Phi) \right. \\ \left. - \frac{a_3 C^2}{48} (D^2 \Phi) (D^2 \Phi) \Phi + \frac{a_4 C^2}{8} \Phi (D^2 \Phi) \Phi^+ + c.c. \right\}, \end{aligned} \quad (3.19)$$

with  $f(x) \frac{1}{\square} g(x) = f(x) \int d^4y G(x-y) g(y)$ . Following the idea of the background field method, we split the chiral and antichiral superfields into their classical and quantum parts

$$\Phi \rightarrow \Phi + \Phi_q, \quad \Phi^+ \rightarrow \Phi^+ + \Phi_q^+ \quad (3.20)$$

and integrate over the quantum superfields in the path integral.

Since  $\Phi_q$  and  $\Phi_q^+$  are chiral and antichiral fields, they are constrained by

$$\bar{D}_{\dot{\alpha}} \Phi_q = D_{\alpha} \Phi_q^+ = 0.$$

One can introduce unconstrained superfields  $\Sigma$  and  $\Sigma^+$  such that

$$\begin{aligned} \Phi_q &= -\frac{1}{4} \bar{D}^2 \Sigma \\ \Phi_q^+ &= -\frac{1}{4} D^2 \Sigma^+. \end{aligned} \quad (3.21)$$

Note that we do not express the background superfields  $\Phi$  and  $\Phi^+$  in terms of  $\Sigma$  and  $\Sigma^+$ , only the quantum parts  $\Phi_q$  and  $\Phi_q^+$ . After integrating over the quantum superfields, the result will be expressed in terms of the (anti)chiral superfields. This

is a big advantage of the background field method (and the supergraph technique). From (3.21) we see that the unconstrained superfields are determined up to a gauge transformation

$$\Sigma \rightarrow \Sigma + \bar{D}_{\dot{\alpha}} \bar{\Lambda}^{\dot{\alpha}} \quad (3.22)$$

$$\Sigma^+ \rightarrow \Sigma^+ + D^{\alpha} \Lambda_{\alpha}, \quad (3.23)$$

where  $\Lambda$  is the gauge parameter. In order to fix this symmetry we have to add a gauge fixing term to the action. For the gauge functions we choose

$$\chi_{\alpha} = D_{\alpha} \Sigma \quad (3.24)$$

$$\bar{\chi}_{\dot{\alpha}} = \bar{D}_{\dot{\alpha}} \Sigma^+. \quad (3.25)$$

The product  $\delta(\chi)\delta(\bar{\chi})$  in the path integral is averaged by the weight  $e^{-i\xi \int d^8z \bar{f} M f}$ :

$$\int df d\bar{f} \delta(\chi_{\alpha} - f_{\alpha}) \delta(\bar{\chi}_{\dot{\alpha}} - \bar{f}_{\dot{\alpha}}) e^{-i\xi \int d^8z \bar{f}^{\dot{\alpha}} M_{\dot{\alpha}\alpha} f^{\alpha}} \quad (3.26)$$

where

$$\bar{f}^{\dot{\alpha}} M_{\dot{\alpha}\alpha} f^{\alpha} = \frac{1}{4} \bar{f}^{\dot{\alpha}} (D_{\alpha} \bar{D}_{\dot{\alpha}} + \frac{3}{4} \bar{D}_{\dot{\alpha}} D_{\alpha}) f^{\alpha} \quad (3.27)$$

and the gauge fixing parameter is denoted by  $\xi$ . The gauge fixing term becomes

$$S_{gf} = -\xi \int d^8z (\bar{D}_{\dot{\alpha}} \bar{\Sigma}) (\frac{3}{16} \bar{D}^{\dot{\alpha}} D^{\alpha} + \frac{1}{4} D^{\alpha} \bar{D}^{\dot{\alpha}}) (D_{\alpha} \Sigma). \quad (3.28)$$

It is easy to show that the ghost fields are decoupled.

The part of the classical gauge fixed action quadratic in quantum superfields is

$$S^{(2)} = \frac{1}{2} \int d^8z \begin{pmatrix} \Sigma & \Sigma^+ \end{pmatrix} (\mathcal{M} + \mathcal{V}) \begin{pmatrix} \Sigma \\ \Sigma^+ \end{pmatrix} \quad (3.29)$$

where the kinetic and the interaction terms are collected in the matrices  $\mathcal{M}$  and  $\mathcal{V}$  respectively. Matrix  $\mathcal{M}$  is given by

$$\mathcal{M} = \begin{pmatrix} -m\Box^{1/2}(1 - a_1 C^2 \Box) P_- & \Box(P_2 + \xi(P_1 + P_T)) \\ \Box(P_1 + \xi(P_2 + P_T)) & -m\Box^{1/2}(1 - a_1 \bar{C}^2 \Box) P_+ \end{pmatrix}. \quad (3.30)$$

Matrix  $\mathcal{V}$  has the form

$$\mathcal{V} = \begin{pmatrix} F & G \\ \bar{G} & \bar{F} \end{pmatrix}, \quad (3.31)$$

with matrix elements

$$\begin{aligned} F &= -\frac{\lambda}{2} \Phi \bar{D}^2 + \frac{\lambda a_2 C^2}{2} \Phi \Box \bar{D}^2 + \frac{\lambda a_2 C^2}{4} (\Box \Phi) \bar{D}^2 \\ &\quad - \frac{\lambda a_3 C^2}{128} \bar{D}^2 (D^2 \Phi) D^2 \bar{D}^2 + \frac{\lambda a_4 C^2}{64} \bar{D}^2 (\Phi^+) D^2 \bar{D}^2, \end{aligned}$$

$$\begin{aligned}
G &= \frac{\lambda a_4 C^2}{64} \overleftarrow{D}^2 (D^2 \Phi) D^2 + \frac{\lambda a_4 \bar{C}^2}{64} \overleftarrow{D}^2 (\bar{D}^2 \Phi^+) D^2, \\
\bar{F} &= -\frac{\lambda}{2} \Phi^+ D^2 + \frac{\lambda a_2 \bar{C}^2}{2} \Phi^+ \square D^2 + \frac{\lambda a_2 \bar{C}^2}{4} (\square \Phi^+) D^2 \\
&\quad - \frac{\lambda a_3 \bar{C}^2}{128} \overleftarrow{D}^2 (\bar{D}^2 \Phi^+) \bar{D}^2 D^2 + \frac{\lambda a_4 \bar{C}^2}{64} \overleftarrow{D}^2 (\Phi) \bar{D}^2 D^2, \\
\bar{G} &= \frac{\lambda a_4 \bar{C}^2}{64} \overleftarrow{D}^2 (\bar{D}^2 \Phi^+) \bar{D}^2 + \frac{\lambda a_4 C^2}{64} \overleftarrow{D}^2 (D^2 \Phi) \bar{D}^2.
\end{aligned} \tag{3.32}$$

The one-loop effective action is

$$\Gamma = S_0 + S_{int} + \frac{i}{2} \text{Tr} \log(1 + \mathcal{M}^{-1} \mathcal{V}). \tag{3.33}$$

The last term in (3.33) is the one-loop correction to the effective action. In order to calculate it we have to invert  $\mathcal{M}$  [16],

$$\mathcal{M}^{-1} = \begin{pmatrix} A & B \\ \bar{B} & \bar{A} \end{pmatrix} = \begin{pmatrix} \frac{m(1-a_1 \bar{C}^2 \square) D^2}{4 \square f(\square)} & \frac{D^2 \bar{D}^2}{16 \square f(\square)} + \frac{\bar{D}^2 D^2 - 2 \bar{D} D^2 \bar{D}}{16 \xi \square^2} \\ \frac{\bar{D}^2 D^2}{16 \square f(\square)} + \frac{D^2 \bar{D}^2 - 2 D \bar{D}^2 D}{16 \xi \square^2} & \frac{m(1-a_1 C^2 \square) \bar{D}^2}{4 \square f(\square)} \end{pmatrix} \tag{3.34}$$

where

$$f(\square) = \square - m^2 + m^2 a_1 (C^2 + \bar{C}^2) \square. \tag{3.35}$$

Expansion of the logarithm in (3.33) gives the one-loop correction to the effective action

$$\Gamma_1 = \frac{i}{2} \text{Tr} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} (\mathcal{M}^{-1} \mathcal{V})^n = \sum_{n=1}^{\infty} \Gamma_1^{(n)}. \tag{3.36}$$

## 4 Results: two-point and three-point Green functions

Let us now look at the divergent parts of our effective action. The first term in the expansion (3.36) is given by

$$\Gamma_1^{(1)} = \frac{i}{2} \text{Tr}(\mathcal{M}^{-1} \mathcal{V}) = \frac{i}{2} \text{Tr} [A F + \bar{A} \bar{F} + B \bar{G} + \bar{B} G] = 0. \tag{4.37}$$

This means that the tadpole contribution to the one-loop effective action vanishes just as in the undeformed Wess-Zumino model.

The second term in (3.36) contains two classical fields and gives the one-loop divergent part of the two-point function. It is given by

$$\begin{aligned}
\Gamma_1^{(2)} &= -\frac{i}{4} \text{Tr}(\mathcal{M}^{-1} \mathcal{V})^2 \\
&= -\frac{i}{4} \text{Tr} [A F A F + \bar{A} \bar{F} \bar{A} \bar{F} + 2 A F B \bar{G} + 2 \bar{A} \bar{F} \bar{B} G \\
&\quad + 2 A G \bar{B} F + 2 \bar{A} \bar{G} B \bar{F} + 2 B \bar{F} \bar{B} F].
\end{aligned} \tag{4.38}$$



In our previous paper [7] we obtained the result for the divergent part of the two-point function applying the background field method to the action written in terms of the component fields and assuming that  $C^2 = \bar{C}^2$ . Here we do not make this assumption. Terms in the classical action (3.18)-(3.19) proportional to  $a_2$  give divergences which cannot be canceled. Therefore, from now on we set  $a_2 = 0$ . Note that this is in agreement with [7].

Divergent parts of the terms appearing in (4.38) are given by

$$\text{Tr}(AFAF)\Big|_{dp} = \frac{im^2\lambda^2 C^2}{4\pi^2\varepsilon} \int d^8z \Phi(z) \left[ a_3 D^2 \Phi(z) - 2a_4 \Phi^+(z) \right], \quad (4.39)$$

$$\text{Tr}(\bar{A}\bar{F}\bar{A}\bar{F})\Big|_{dp} = \frac{im^2\lambda^2 \bar{C}^2}{4\pi^2\varepsilon} \int d^8z \Phi^+(z) \left[ a_3 \bar{D}^2 \Phi^+(z) - 2a_4 \Phi(z) \right], \quad (4.40)$$

$$\begin{aligned} \text{Tr}(AFB\bar{G})\Big|_{dp} &= \text{Tr}(AG\bar{B}F)\Big|_{dp} \\ &= -\frac{im\lambda^2 C^2 a_4}{16\pi^2\varepsilon} \int d^8z \Phi(z) D^2 \Phi(z), \end{aligned} \quad (4.41)$$

$$\begin{aligned} \text{Tr}(\bar{A}\bar{F}\bar{B}G)\Big|_{dp} &= \text{Tr}(\bar{A}\bar{G}\bar{B}\bar{F})\Big|_{dp} \\ &= -\frac{im\lambda^2 \bar{C}^2 a_4}{16\pi^2\varepsilon} \int d^8z \Phi^+(z) \bar{D}^2 \Phi^+(z), \end{aligned} \quad (4.42)$$

$$\text{Tr}(B\bar{F}\bar{B}F)\Big|_{dp} = \frac{i\lambda^2(1 - 2a_1 m^2(C^2 + \bar{C}^2))}{2\pi^2\varepsilon} \int d^8z \Phi^+(z) \Phi(z). \quad (4.43)$$

From (4.39-4.43) and (4.38) we obtain

$$\begin{aligned} \Gamma_1^{(2)}\Big|_{dp} &= \frac{\lambda^2(1 - (2a_1 + \frac{a_4}{2})m^2(C^2 + \bar{C}^2))}{4\pi^2\varepsilon} \int d^8z \Phi^+(z) \Phi(z) \\ &\quad + \frac{\lambda^2 m(ma_3 - a_4)}{16\pi^2\varepsilon} \int d^8z [C^2 \Phi(z) D^2 \Phi(z) + c.c.]. \end{aligned} \quad (4.44)$$

This result can be easily rewritten in terms of the component fields and it is in agreement with the result obtained in [7]. Note that from equation (4.44) no conditions follow on the coefficients  $a_1$ ,  $a_3$  and  $a_4$ : they can be arbitrary.

Let us consider the divergent part of the three-point Green function

$$\begin{aligned} \Gamma_1^{(3)} &= \frac{i}{6} \text{Tr}(\mathcal{M}^{-1} \mathcal{V})^3 \\ &= \frac{i}{6} \text{Tr} \left[ AF AF AF + \bar{A}\bar{F} \bar{A}\bar{F} \bar{A}\bar{F} + 3AF AFB\bar{G} + 3\bar{A}\bar{F} \bar{A}\bar{F} \bar{B}G + 3AF AG\bar{B}\bar{F} \right. \\ &\quad + 3\bar{A}\bar{F} \bar{A}\bar{G}B\bar{F} + 3AFB\bar{F}\bar{B}F + 3\bar{A}\bar{F}\bar{B}\bar{F}B\bar{F} + 3AFB\bar{F}\bar{A}\bar{G} \\ &\quad \left. + 3\bar{A}\bar{F}\bar{B}\bar{F}AG + 3B\bar{G}B\bar{F}\bar{B}F + 3\bar{B}G\bar{B}\bar{F}B\bar{F} \right]. \end{aligned} \quad (4.45)$$

The traces appearing in (4.45) are

$$\begin{aligned} \text{Tr}(AFB\bar{F}\bar{B}F) \Big|_{dp} &= \frac{im\lambda^3 a_1 \bar{C}^2}{\pi^2 \varepsilon} \int d^8 z \Phi(z) \Phi(z) \Phi^+(z) \\ &\quad - \frac{im\lambda^3 a_3 C^2}{2\pi^2 \varepsilon} \int d^8 z \Phi(z) \Phi^+(z) D^2 \Phi(z) \\ &\quad + \frac{im\lambda^3 a_4 C^2}{\pi^2 \varepsilon} \int d^8 z \Phi(z) \Phi^+(z) \Phi^+(z), \end{aligned} \quad (4.46)$$

$$\begin{aligned} \text{Tr}(\bar{A}\bar{F}\bar{B}F\bar{B}\bar{F}) \Big|_{dp} &= \frac{im\lambda^3 a_1 C^2}{\pi^2 \varepsilon} \int d^8 z \Phi^+(z) \Phi^+(z) \Phi(z) \\ &\quad - \frac{im\lambda^3 a_3 \bar{C}^2}{2\pi^2 \varepsilon} \int d^8 z \Phi^+(z) \Phi(z) \bar{D}^2 \Phi^+(z) \\ &\quad + \frac{im\lambda^3 a_4 \bar{C}^2}{\pi^2 \varepsilon} \int d^8 z \Phi^+(z) \Phi(z) \Phi(z), \end{aligned} \quad (4.47)$$

$$\begin{aligned} \text{Tr}(B\bar{G}B\bar{F}\bar{B}F) \Big|_{dp} &= \text{Tr}(\bar{B}\bar{G}\bar{B}F\bar{B}\bar{F}) \Big|_{dp} \\ &= \frac{i\lambda^3 a_4}{8\pi^2 \varepsilon} \int d^8 z \left[ C^2 \Phi(z) \Phi^+(z) D^2 \Phi(z) + c.c. \right]. \end{aligned} \quad (4.48)$$

All other terms appearing in (4.45) are convergent. We obtain

$$\begin{aligned} \Gamma_1^{(3)} \Big|_{dp} &= -\frac{m\lambda^3(a_1 + a_4)}{2\pi^2 \varepsilon} \int d^8 z \left[ \bar{C}^2 \Phi(z) \Phi(z) \Phi^+(z) + c.c. \right] \\ &\quad + \frac{\lambda^3(2ma_3 - a_4)}{8\pi^2 \varepsilon} \int d^8 z \left[ C^2 \Phi(z) \Phi^+(z) D^2 \Phi(z) + c.c. \right]. \end{aligned} \quad (4.49)$$

There is a term in the first line of (4.49) which is not present in the classical action (3.18)-(3.19). In order to have a renormalizable model, we have to set  $a_1 + a_4 = 0$ ;  $a_3$  remains arbitrary. The second term in (4.49) corresponds to the " $a_4$ -term" in the classical action (3.18)-(3.19). Note that in the model with  $a_4 = 0$  we would not be able to cancel divergences in the three-point functions, which was our motivation to introduce this term in the classical action from the very beginning.

## 5 Results: four-point Green function

Before making any final statement about renormalizability of our model we still have to check if divergences in the four-point function can be cancelled. To this end, let us consider the four-point function. It is given by

$$\begin{aligned} \Gamma_1^{(4)} &= -\frac{i}{8} \text{Tr}(\mathcal{M}^{-1} \mathcal{V})^4 \\ &= -\frac{i}{8} \text{Tr} \left[ AFAFAFAF + \bar{A}\bar{F}\bar{A}\bar{F}\bar{A}\bar{F}\bar{A}\bar{F} + 2B\bar{F}\bar{B}FB\bar{F}\bar{B}F \right] \end{aligned}$$

$$\begin{aligned}
& -\frac{i}{2}\text{Tr}\left[AFAFB\bar{F}\bar{B}F + \bar{A}\bar{F}\bar{A}\bar{F}\bar{B}F\bar{B}F + AFB\bar{F}\bar{A}\bar{F}\bar{B}F + AFAFAFB\bar{G} \right. \\
& + \bar{A}\bar{F}\bar{A}\bar{F}\bar{A}\bar{F}\bar{B}G + AFAFB\bar{F}\bar{A}\bar{G} + \bar{A}\bar{F}\bar{A}\bar{F}\bar{B}FAG + AFAFAG\bar{B}F \\
& + \bar{A}\bar{F}\bar{A}\bar{F}\bar{A}\bar{G}B\bar{F} + AFB\bar{G}B\bar{F}\bar{B}F + \bar{A}\bar{F}\bar{B}G\bar{B}F\bar{B}F + AFB\bar{F}\bar{B}F\bar{B}G \\
& + \bar{A}\bar{F}\bar{B}F\bar{B}F\bar{B}G + AFB\bar{F}\bar{A}\bar{F}\bar{A}\bar{G} + \bar{A}\bar{F}\bar{B}F\bar{A}FAG + AFB\bar{F}\bar{B}G\bar{B}F \\
& \left. + \bar{A}\bar{F}\bar{B}F\bar{B}G\bar{B}F + B\bar{F}\bar{B}F\bar{B}F\bar{A}\bar{G} + \bar{B}F\bar{B}F\bar{B}FAG\right]. \quad (5.50)
\end{aligned}$$

There is only one non-vanishing divergent term in (5.50) and it is given by

$$\text{Tr}(B\bar{F}\bar{B}F\bar{B}F\bar{B}F)\Big|_{dp} = \frac{i\lambda^4}{\pi^2\epsilon} \int d^8z \left[ \bar{C}^2\Phi(z)\Phi(z)\Phi^+(z) \left( a_3\bar{D}^2\Phi^+(z) - 2a_4\Phi(z) \right) + c.c. \right]. \quad (5.51)$$

Therefore the divergent part of the four-point function does not vanish and it is given by

$$\Gamma_1^{(4)}\Big|_{dp} = \frac{\lambda^4}{8\pi^2\epsilon} \int d^8z \left[ \bar{C}^2\Phi(z)\Phi(z)\Phi^+(z) \left( a_3\bar{D}^2\Phi^+(z) - 2a_4\Phi(z) \right) + c.c. \right]. \quad (5.52)$$

We see that this term does not appear in the classical action (2.17) and we therefore cannot cancel divergences in the four-point function.

Since the divergent terms appearing in the three-point function (4.49) have the same form as (3.19), one could naively conclude that the three-point function is renormalizable and that only the four-point divergences (5.52) spoil renormalizability. However, to prove this we have to redefine the fields and the coupling constants explicitly. To cancel divergences we add counterterms to the classical action (3.18), (3.19). The bare action is given by

$$S_B = S_0 + S_{int} - \Gamma_1^{(2)}\Big|_{dp} - \Gamma_1^{(3)}\Big|_{dp}. \quad (5.53)$$

The two-point Green function in (5.53) gives the renormalization of the superfield  $\Phi$

$$\Phi_0 = \sqrt{Z}\Phi, \quad (5.54)$$

where

$$Z = 1 - \frac{\lambda^2}{4\pi^2\epsilon} \left( 1 - \frac{3}{2}a_1m^2(C^2 + \bar{C}^2) \right) \quad (5.55)$$

and

$$m = Zm_0 \quad (5.56)$$

since  $\delta_m = 0$ . In addition to the field redefinition we get the redefinition of the coupling constants

$$a_{10}C_0^2 = a_1C^2 \left( 1 - \frac{\lambda^2(ma_3 + a_1)}{2\pi^2\epsilon a_1} \right). \quad (5.57)$$

From the three-point Green function in (5.53) we obtain the following conditions

$$\begin{aligned}\lambda &= Z^{3/2}\lambda_0, \\ a_{10}C_0^2 &= a_1C^2\left(1 + \frac{\lambda^2(2ma_3 + a_1)}{\pi^2\epsilon a_1}\right),\end{aligned}\tag{5.58}$$

$$a_{30}C_0^2 = a_3C^2.\tag{5.59}$$

It is obvious that results (5.57), (5.58) and (5.59) are not compatible. We therefore conclude that all three-point counterterms are present in the classical action (3.18)-(3.19) but there is not enough parameters in the theory, that is, fields and coupling constants, to absorb all divergences. In addition, the divergent part (5.52) of the four-point function is not present in the classical action. This situation is very similar to the one in the noncommutative gauge theory with a massless Dirac fermion  $\psi$  [18], where only two-point Green functions were renormalizable. In that model the four- $\psi$  divergence appears and it cannot be removed.

## 6 Discussion and conclusions

In order to see how a deformation by twist of the Wess-Zumino model affects renormalizability properties of the model, we consider a special example of the twist, (2.6). Compared with the classical SUSY Hopf algebra, the twisted SUSY Hopf algebra is unchanged. In particular, the Leibniz rule (2.14) remains undeformed. The notion of chirality is however lost and we have to apply the method of projectors introduced in [8] to obtain the action. By including all terms invariant under the deformed SUSY transformation, we formulate a deformation of the usual Wess-Zumino action (2.17) and discuss its renormalizability.

Using the background field method and the supergraph technique we calculate the divergent parts of the two-point, three-point and four-point functions up to second order in the deformation parameter  $C_{\alpha\beta}$ . For one-point and two-point functions the obtained results are in agreement with the results in [7], where the analysis was done in component fields. There is no tadpole diagram, no mass renormalization and all fields are renormalized in the same way. In an attempt to obtain a renormalizable three-point function we add an additional term to the classical action: the new term is invariant under the deformed SUSY transformations (2.13) and its commutative limit is zero. Therefore we obtain the action parametrized by three coefficients  $a_1$ ,  $a_3$ ,  $a_4$ . These coefficients are partially fixed by the requirement that the three point function be renormalizable. Unfortunately, divergences in the four-point function cannot be absorbed in this manner. In addition, even on the level of the three-point function in fact there is not enough parameters to make a redefinition of fields and coupling constants consistently. Therefore we conclude that our model (2.17) is not renormalizable.

Having in mind results of [19] we also investigated on-shell renormalizability of our model. In general, on-shell renormalizability leads to a one-loop renormalizable  $S$ -matrix. On the other hand, one-loop renormalizable Green functions spoil renormalizability at higher loops. After using the equations of motion to obtain on-shell

divergent terms in our model, we are left with some non-vanishing five-point divergences. Since the five-point Green function is convergent, these five-point divergences cannot be canceled. Therefore we have to conclude that our model is not on-shell renormalizable.

Renormalization of the deformed Wess-Zumino model has been previously studied in the literature, see for example [20], [21]. The models considered there are non-hermitian and have half of the supersymmetry of the corresponding  $\mathcal{N} = 1$  theory. In general they are not power-counting renormalizable, however it has been argued in [20], [21] that they are nevertheless renormalizable since only a finite number of additional terms needs to be added to the action to absorb divergences to all orders. This fact is related to the non-hermiticity of the relevant actions. Renormalizability of the deformed Wess-Zumino model in [20] was achieved by adding a finite number of additional terms to the original classical action and using the equations of motion to eliminate auxiliary field  $F$  from the theory.

The models [20], [21] are different from the model we study here. The main difference lies in the deformation, resulting in a different  $\star$ -product and a different deformed action. The action of our model is hermitian<sup>3</sup> and moreover, invariant under the full  $\mathcal{N} = 1$  SUSY. Having in mind the results of [21] we checked renormalizability of the non-hermitian action (3.18)-(3.19) omitting the complex conjugate terms. The result was the same, the model (as it stands) is not renormalizable. Since our model is more complicated than those discussed in [20] it is not obvious which terms should be added in order to cancel all one-loop divergences, and whether the number of the added terms is finite. This remains to be investigated in future. Another possibility is to check whether a different deformation would give a better behaving model. In that way, one could say which deformation/non(anti)commutativity would be preferred.

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<sup>3</sup>The deformation (2.9) is not hermitian. However, by adding the complex conjugate terms by hand we obtain the hermitian action (3.18)-(3.19).

## A Calculation of traces

In this appendix we give calculations of some of the traces appearing in  $\Gamma_1^{(3)}$  and  $\Gamma_1^{(4)}$ . Up to second order in the deformation parameter the trace  $\text{Tr}(AFB\bar{F}\bar{B}F)$  is given by

$$\begin{aligned}
\text{Tr}(AFB\bar{F}\bar{B}F) &= \\
&= \int \prod_{i=1}^4 d^8 z_i \left( \frac{m(1 - a_1 \bar{C}^2 \square) D^2}{4 \square (\square - m^2)} \right)_1 \delta(z_1 - z_2) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_2 \\
&\quad \cdot \left( \frac{D^2 \bar{D}^2}{16 \square (\square - m^2)} \right)_2 \delta(z_2 - z_3) \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_3 \\
&\quad \cdot \left( \frac{\bar{D}^2 D^2}{16 \square (\square - m^2)} \right)_3 \delta(z_3 - z_4) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_4 \delta(z_4 - z_1) \\
&+ \int \prod_{i=1}^4 d^8 z_i \left( \frac{m D^2}{4 \square (m^2 - \square)} \right)_1 \delta(z_1 - z_2) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_2 \\
&\quad \cdot \left( \frac{D^2 \bar{D}^2}{16 \square (m^2 - \square)} \right)_2 \delta(z_2 - z_3) \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_3 \\
&\quad \cdot \left( \frac{\bar{D}^2 D^2}{16 \square (m^2 - \square)} \right)_3 \delta(z_3 - z_4) \\
&\quad \cdot \left[ \bar{D}^2 \left( -\frac{\lambda C^2}{128} (a_3 D^2 \Phi - 2a_4 \Phi^+) \right) D^2 \bar{D}^2 \right]_4 \delta(z_4 - z_1) \\
&+ \int \prod_{i=1}^4 d^8 z_i \left( \frac{m D^2}{4 \square (m^2 - \square)} \right)_1 \delta(z_1 - z_2) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_2 \\
&\quad \cdot \left( \frac{D^2 \bar{D}^2}{16 \square (m^2 - \square)} \right)_2 \delta(z_2 - z_3) \left[ \bar{D}^2 \left( -\frac{\lambda \bar{C}^2}{128} (a_3 \bar{D}^2 \Phi^+ - 2a_4 \Phi) \right) \bar{D}^2 D^2 \right]_3 \\
&\quad \cdot \left( \frac{\bar{D}^2 D^2}{16 \square (m^2 - \square)} \right)_3 \delta(z_3 - z_4) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_4 \delta(z_4 - z_1) \\
&+ \int \prod_{i=1}^4 d^8 z_i \left( \frac{m D^2}{4 \square (m^2 - \square)} \right)_1 \delta(z_1 - z_2) \\
&\quad \cdot \left[ \bar{D}^2 \left( -\frac{\lambda C^2}{128} (a_3 D^2 \Phi - 2a_4 \Phi^+) \right) D^2 \bar{D}^2 \right]_2 \left( \frac{D^2 \bar{D}^2}{16 \square (m^2 - \square)} \right)_2 \delta(z_2 - z_3) \\
&\quad \cdot \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_3 \left( \frac{\bar{D}^2 D^2}{16 \square (m^2 - \square)} \right)_3 \delta(z_3 - z_4) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_4 \delta(z_4 - z_1) .
\end{aligned}$$

After applying the  $D$ -algebra identities, we obtain

$$\begin{aligned}
\text{Tr}(AFB\bar{F}\bar{B}F) &= \\
&= \int d^4 \theta \int \prod_{i=1}^3 d^8 x_i (-8m\lambda^3) \left( \frac{1 - a_1 \bar{C}^2 \square}{\square - m^2} \right)_1 \delta(x_1 - x_2) \Phi(x_2, \theta) \\
&\quad \cdot \left( \frac{1}{\square - m^2} \right)_2 \delta(x_2 - x_3) \Phi^+(x_3, \theta) \left( \frac{1}{\square - m^2} \right)_3 \delta(x_3 - x_1) \Phi(x_1, \theta)
\end{aligned}$$

$$\begin{aligned}
& + \int d^4\theta \int \prod_{i=1}^3 d^8x_i (-2m\lambda^3 C^2) \left( \frac{1}{\square - m^2} \right)_1 \delta(x_1 - x_2) \Phi(x_2, \theta) \\
& \quad \cdot \left( \frac{1}{\square - m^2} \right)_2 \delta(x_2 - x_3) \Phi^+(x_3, \theta) \left( \frac{\square}{\square - m^2} \right)_3 \delta(x_3 - x_1) \\
& \quad \cdot \left( a_3 D^2 \Phi(x_1, \theta) - 2a_4 \Phi^+(x_1, \theta) \right) \\
& + \int d^4\theta \int \prod_{i=1}^3 d^8x_i (-2m\lambda^3 \bar{C}^2) \left( \frac{1}{\square - m^2} \right)_1 \delta(x_1 - x_2) \Phi(x_2, \theta) \\
& \quad \cdot \left( \frac{1}{\square - m^2} \right)_2 \delta(x_2 - x_3) \left( a_3 \bar{D}^2 \Phi^+(x_3, \theta) - 2a_4 \Phi(x_3, \theta) \right) \\
& \quad \cdot \left( \frac{\square}{\square - m^2} \right)_3 \delta(x_3 - x_1) \Phi(x_1, \theta) \\
& + \int d^4\theta \int \prod_{i=1}^3 d^8x_i (-2m\lambda^3 C^2) \left( \frac{1}{\square - m^2} \right)_1 \delta(x_1 - x_2) \\
& \quad \cdot \left( a_3 D^2 \Phi(x_2, \theta) - 2a_4 \Phi^+(x_2, \theta) \right) \left( \frac{\square}{\square - m^2} \right)_2 \delta(x_2 - x_3) \\
& \quad \cdot \Phi^+(x_3, \theta) \left( \frac{1}{\square - m^2} \right)_3 \delta(x_3 - x_1) \Phi(x_1, \theta) . \tag{1.60}
\end{aligned}$$

Transforming the previous expression to the momentum space and using the dimensional regularization we obtain (4.46).

Similarly, we find

$$\begin{aligned}
& \text{Tr}(B\bar{G}B\bar{F}\bar{B}F) = \\
& = \int \prod_{i=1}^4 d^8z_i \left( \frac{D^2 \bar{D}^2}{16\square(\square - m^2)} \right)_1 \delta(z_1 - z_2) \left[ \bar{D}^2 \left( \frac{\lambda a_4}{64} (\bar{C}^2 \bar{D}^2 \Phi^+ + C^2 D^2 \Phi) \right) \bar{D}^2 \right]_2 \\
& \quad \cdot \left( \frac{D^2 \bar{D}^2}{16\square(\square - m^2)} \right)_2 \delta(z_2 - z_3) \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_3 \\
& \quad \cdot \left( \frac{\bar{D}^2 D^2}{16\square(\square - m^2)} \right)_3 \delta(z_3 - z_4) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_4 \delta(z_4 - z_1) \\
& = \int d^4\theta \int \prod_{i=1}^3 d^8x_i (\lambda^3 \bar{C}^2 a_4) \left( \frac{\square}{\square - m^2} \right)_1 \delta(x_1 - x_2) \bar{D}^2 \Phi^+(x_2, \theta) \\
& \quad \cdot \left( \frac{1}{\square - m^2} \right)_2 \delta(x_2 - x_3) \Phi^+(x_3, \theta) \left( \frac{1}{\square - m^2} \right)_3 \delta(x_3 - x_1) \Phi(x_1, \theta) \\
& + \int d^4\theta \int \prod_{i=1}^3 d^8x_i (\lambda^3 C^2 a_4) \left( \frac{1}{\square - m^2} \right)_1 \delta(x_1 - x_2) D^2 \Phi(x_2, \theta) \\
& \quad \cdot \left( \frac{\square}{\square - m^2} \right)_2 \delta(x_2 - x_3) \Phi^+(x_3, \theta) \left( \frac{1}{\square - m^2} \right)_3 \delta(x_3 - x_1) \Phi(x_1, \theta) . \tag{1.61}
\end{aligned}$$

The divergent part of (1.61) is (4.48).

The term contributing to the divergent part of the four-point Green function is

$$\begin{aligned}
& \text{Tr}(B\bar{F}\bar{B}FB\bar{F}\bar{B}F) = \\
& = \int \prod_{i=1}^5 d^8 z_i \left( \frac{D^2 \bar{D}^2}{16\Box(\Box - m^2)} \right)_1 \delta(z_1 - z_2) \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_2 \\
& \quad \cdot \left( \frac{\bar{D}^2 D^2}{16\Box(\Box - m^2)} \right)_2 \delta(z_2 - z_3) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_3 \\
& \quad \cdot \left( \frac{D^2 \bar{D}^2}{16\Box(\Box - m^2)} \right)_3 \delta(z_3 - z_4) \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_4 \\
& \quad \cdot \left( \frac{\bar{D}^2 D^2}{16\Box(\Box - m^2)} \right)_4 \delta(z_4 - z_5) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_5 \delta(z_5 - z_1) \\
& + 2 \int \prod_{i=1}^5 d^8 z_i \left( \frac{D^2 \bar{D}^2}{16\Box(\Box - m^2)} \right)_1 \delta(z_1 - z_2) \\
& \quad \cdot \left[ \bar{D}^2 \left( -\frac{\lambda \bar{C}^2}{128} (a_3 \bar{D}^2 \Phi^+ - 2a_4 \Phi) \right) \bar{D}^2 D^2 \right]_2 \\
& \quad \cdot \left( \frac{\bar{D}^2 D^2}{16\Box(\Box - m^2)} \right)_2 \delta(z_2 - z_3) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_3 \left( \frac{D^2 \bar{D}^2}{16\Box(\Box - m^2)} \right)_3 \delta(z_3 - z_4) \\
& \quad \cdot \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_4 \left( \frac{\bar{D}^2 D^2}{16\Box(\Box - m^2)} \right)_4 \delta(z_4 - z_5) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_5 \delta(z_5 - z_1) \\
& + 2 \int \prod_{i=1}^5 d^8 z_i \left( \frac{\bar{D}^2 D^2}{16\Box(\Box - m^2)} \right)_1 \delta(z_1 - z_2) \\
& \quad \cdot \left[ \bar{D}^2 \left( -\frac{\lambda C^2}{128} (a_3 D^2 \Phi - 2a_4 \Phi^+) \right) D^2 \bar{D}^2 \right]_2 \\
& \quad \cdot \left( \frac{D^2 \bar{D}^2}{16\Box(\Box - m^2)} \right)_2 \delta(z_2 - z_3) \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_3 \left( \frac{\bar{D}^2 D^2}{16\Box(\Box - m^2)} \right)_3 \delta(z_3 - z_4) \\
& \quad \cdot \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_4 \left( \frac{D^2 \bar{D}^2}{16\Box(\Box - m^2)} \right)_4 \delta(z_4 - z_5) \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_5 \delta(z_5 - z_1) \\
& = \int \prod_{i=1}^4 d^8 z_i (16\lambda^4) \left( \frac{1}{\Box - m^2} \right)_1 e^{i(\theta_1 \sigma^n \bar{\theta}_1 + \theta_2 \sigma^n \bar{\theta}_2 - 2\theta_1 \sigma^n \bar{\theta}_2) \partial_n^1} \delta(x_1 - x_2) \\
& \quad \cdot \Phi^+(z_2) \left( \frac{1}{\Box - m^2} \right)_2 \delta(z_2 - z_3) \Phi(z_3) \\
& \quad \cdot \left( \frac{1}{\Box - m^2} \right)_3 e^{i(\theta_3 \sigma^n \bar{\theta}_3 + \theta_4 \sigma^n \bar{\theta}_4 - 2\theta_3 \sigma^n \bar{\theta}_4) \partial_n^3} \delta(x_3 - x_4) \Phi^+(z_4) \\
& \quad \cdot \left( \frac{1}{\Box - m^2} \right)_4 \delta(z_4 - z_1) \Phi(z_1) \\
& + \int d^4 \theta \int \prod_{i=1}^4 d^8 x_i (8\lambda^4 \bar{C}^2) \left( \frac{\Box}{\Box - m^2} \right)_1 \delta(x_1 - x_2)
\end{aligned}$$



$$\begin{aligned}
& \cdot (a_3 \bar{D}^2 \Phi^+(x_2, \theta) - 2a_4 \Phi(x_2, \theta)) \left( \frac{\square}{\square - m^2} \right)_2 \delta(x_2 - x_3) \Phi(x_3, \theta) \\
& \cdot \left( \frac{1}{\square - m^2} \right)_3 \delta(x_3 - x_4) \Phi^+(x_4, \theta) \left( \frac{1}{\square - m^2} \right)_4 \delta(x_4 - x_1) \Phi(x_1, \theta) \\
& + \int d^4 \theta \int \prod_{i=1}^4 d^8 x_i (8\lambda^4 C^2) \left( \frac{\square}{\square - m^2} \right)_1 \delta(x_1 - x_2) \\
& \cdot (a_3 D^2 \Phi(x_2, \theta) - 2a_4 \Phi^+(x_2, \theta)) \left( \frac{\square}{\square - m^2} \right)_2 \delta(x_2 - x_3) \Phi^+(x_3, \theta) \\
& \cdot \left( \frac{1}{\square - m^2} \right)_3 \delta(x_3 - x_4) \Phi(x_4, \theta) \left( \frac{1}{\square - m^2} \right)_4 \delta(x_4 - x_1) \Phi^+(x_1, \theta) . \quad (1.62)
\end{aligned}$$

From (1.62) the divergence (5.51) follows.

All divergences appearing in (1.60), (1.61) and (1.62) are obtained using the following formulae:

$$\begin{aligned}
& \int \prod_{i=1}^3 d^4 x_i \left( \frac{\square}{\square - m^2} \right)_1 \delta(x_1 - x_2) f_1(x_2) \left( \frac{1}{\square - m^2} \right)_2 \delta(x_2 - x_3) f_2(x_3) \\
& \cdot \left( \frac{1}{\square - m^2} \right)_3 \delta(x_3 - x_1) f_3(x_1) \Big|_{dp} \\
& = \frac{i}{8\pi^2 \varepsilon} \int d^4 x f_1(x) f_2(x) f_3(x), \quad (1.63)
\end{aligned}$$

$$\begin{aligned}
& \int \prod_{i=1}^4 d^4 x_i \left( \frac{\square}{\square - m^2} \right)_1 \delta(x_1 - x_2) f_1(x_2) \left( \frac{\square}{\square - m^2} \right)_2 \delta(x_2 - x_3) f_2(x_3) \\
& \cdot \left( \frac{1}{\square - m^2} \right)_3 \delta(x_3 - x_4) f_3(x_4) \left( \frac{1}{\square - m^2} \right)_4 \delta(x_4 - x_1) f_4(x_1) \Big|_{dp} \\
& = \frac{i}{8\pi^2 \varepsilon} \int d^4 x f_1(x) f_2(x) f_3(x) f_4(x). \quad (1.64)
\end{aligned}$$

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# (Non)renormalizability of the D-deformed Wess-Zumino model

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## Abstract

We continue the analysis of the  $D$ -deformed Wess-Zumino model which we introduced in the previous paper. The model is defined by a deformation which is non-hermitian and given in terms of the covariant derivatives  $D_\alpha$ . We calculate one-loop divergences in the two-point, three-point and four-point Green functions. Possibilities to render the model renormalizable are discussed.

**Keywords:** supersymmetry, non-hermitian twist, deformed Wess-Zumino model, supergraph technique, renormalization

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# 1 Introduction

Having in mind problems which physics encounters at small scales (high energies), in recent years attempts were made to combine supersymmetry (SUSY) with noncommutative geometry. Different models were constructed, see for example [1], [2], [3], [4], [5], [6], [7], [8], [9], [10]. Some of these models emerge naturally as low energy limits of string theories in a background with a constant Neveu-Schwarz two form and/or a constant Ramond-Ramond two form. For some references on noncommutative geometry and non(anti)commutative field theories see [7].

One way to describe a noncommutative deformation is to consider the algebra of functions on a smooth manifold with the usual pointwise multiplication replaced by a noncommutative  $\star$ -product. A wide class of  $\star$ -products can be defined by twists  $\mathcal{F}$ . The notion of twist was first introduced in [11] in the context of quantum groups; recently it has been used to describe symmetries of noncommutative spaces, see for example [6], [12], [13].

In our previous paper [7] we started the analysis of a simple model. Since we are interested in the renormalizability properties of the supersymmetric theories with twisted symmetry, we introduce the non(anti)commutative deformation via the twist

$$\mathcal{F} = e^{\frac{1}{2}C^{\alpha\beta}D_\alpha \otimes D_\beta}. \quad (1.1)$$

Here  $C^{\alpha\beta} = C^{\beta\alpha} \in \mathbb{C}$  is a complex constant matrix and  $D_\alpha = \partial_\alpha + i\sigma_{\alpha\dot{\alpha}}^m \bar{\theta}^{\dot{\alpha}} \partial_m$  are the SUSY covariant derivatives. The twist (1.1) is not hermitian under the usual complex conjugation. Due to our choice of the twist, the coproduct of the SUSY transformations remains undeformed, leading to the undeformed Leibniz rule. The inverse of (1.1) defines the  $\star$ -product. It is obvious that the  $\star$ -product of two chiral fields will not be a chiral field. Therefore we have to use projectors to separate chiral and antichiral parts. The deformed Wess-Zumino action is then constructed by inclusion of all possible invariants under the deformed SUSY transformations.

The plan of the paper is as follows. In the next section we summarize the most important properties of our model, the details of the construction are given in [7]. However, we introduce two additional terms in the action. These terms are SUSY invariant and they vanish in the commutative limit; they were not considered in the previous paper for two reasons. Namely, their presence is not required by the renormalizability of the two-point function; in addition, they represent a non-minimal deformation. In other words, they are a deformation of a term not present in the commutative Wess-Zumino action. We shall see that these new terms are essential in order to obtain a renormalizable model. In Section 3 we describe the method we are using to calculate divergent parts of the  $n$ -point Green functions: the background field method and the supergraph technique. In Sections 4 and 5 the divergent parts of the two-point, three-point and four-point functions are calculated. In Section 6 we discuss renormalizability of the model. In the final section, we give some comments and compare our results with the results already present in the literature. Some details of our calculations are presented in appendix.

## 2 $D$ -deformed Wess-Zumino model

We work in the superspace generated by  $x^m$ ,  $\theta^\alpha$  and  $\bar{\theta}_{\dot{\alpha}}$  coordinates which fulfill

$$[x^m, x^n] = [x^m, \theta^\alpha] = [x^m, \bar{\theta}_{\dot{\alpha}}] = 0, \quad \{\theta^\alpha, \theta^\beta\} = \{\bar{\theta}_{\dot{\alpha}}, \bar{\theta}_{\dot{\beta}}\} = \{\theta^\alpha, \bar{\theta}_{\dot{\alpha}}\} = 0, \quad (2.2)$$

with  $m = 0, \dots, 3$  and  $\alpha, \beta = 1, 2$ . These coordinates we call supercoordinates, to  $x^m$  we refer as bosonic and to  $\theta^\alpha$  and  $\bar{\theta}_{\dot{\alpha}}$  we refer as fermionic coordinates. We work in Minkowski space-time with the metric  $(-, +, +, +)$  and  $x^2 = x^m x_m = -(x^0)^2 + (x^1)^2 + (x^2)^2 + (x^3)^2$ .

A general superfield  $F(x, \theta, \bar{\theta})$  can be expanded in powers of  $\theta$  and  $\bar{\theta}$ ,

$$\begin{aligned} F(x, \theta, \bar{\theta}) = & f(x) + \theta \phi(x) + \bar{\theta} \bar{\chi}(x) + \theta \theta m(x) + \bar{\theta} \bar{\theta} n(x) + \theta \sigma^m \bar{\theta} v_m(x) \\ & + \theta \theta \bar{\theta} \bar{\lambda}(x) + \bar{\theta} \bar{\theta} \theta \varphi(x) + \theta \theta \bar{\theta} \bar{\theta} d(x). \end{aligned} \quad (2.3)$$

Under the infinitesimal SUSY transformations it transforms as

$$\delta_\xi F = (\xi Q + \bar{\xi} \bar{Q}) F, \quad (2.4)$$

where  $\xi^\alpha$  and  $\bar{\xi}_{\dot{\alpha}}$  are constant anticommuting parameters and  $Q^\alpha$  and  $\bar{Q}_{\dot{\alpha}}$  are SUSY generators,

$$Q_\alpha = \partial_\alpha - i \sigma_{\alpha\dot{\alpha}}^m \bar{\theta}^{\dot{\alpha}} \partial_m, \quad \bar{Q}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} + i \theta^\alpha \sigma_{\alpha\dot{\alpha}}^m \partial_m. \quad (2.5)$$

As in [8], [13] we introduce a deformation of the Hopf algebra of infinitesimal SUSY transformations by choosing the twist  $\mathcal{F}$  in the following way

$$\mathcal{F} = e^{\frac{1}{2} C^{\alpha\beta} D_\alpha \otimes D_\beta}, \quad (2.6)$$

with the complex constant matrix  $C^{\alpha\beta} = C^{\beta\alpha} \in \mathbb{C}$ . Note that this twist<sup>1</sup> is not hermitian,  $\mathcal{F}^* \neq \mathcal{F}$ ; the usual complex conjugation is denoted by " $*$ ". It can be shown that (2.6) satisfies all requirements for a twist, [14]. The Hopf algebra of infinitesimal SUSY transformations does not change since

$$\{Q_\alpha, D_\beta\} = \{\bar{Q}_{\dot{\alpha}}, D_\beta\} = 0. \quad (2.7)$$

This means that the full supersymmetry is preserved.

The inverse of the twist (2.6),

$$\mathcal{F}^{-1} = e^{-\frac{1}{2} C^{\alpha\beta} D_\alpha \otimes D_\beta}, \quad (2.8)$$

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<sup>1</sup>Strictly speaking, the twist  $\mathcal{F}$  (2.6) does not belong to the universal enveloping algebra of the Lie algebra of infinitesimal SUSY transformations. Therefore to be mathematically correct we should enlarge the algebra by introducing the relations for the operators  $D_\alpha$  as well. In this way the deformed SUSY Hopf algebra remains the same as the undeformed one. However, since  $[D_\alpha, M_{mn}] \neq 0$  the super Poincaré algebra becomes deformed and different from the super Poincaré algebra in the commutative case.

defines the  $\star$ -product. For arbitrary superfields  $F$  and  $G$  the  $\star$ -product reads

$$\begin{aligned}
F \star G &= \mu_\star \{F \otimes G\} \\
&= \mu \{ \mathcal{F}^{-1} F \otimes G \} \\
&= F \cdot G - \frac{1}{2} (-1)^{|F|} C^{\alpha\beta} (D_\alpha F) \cdot (D_\beta G) \\
&\quad - \frac{1}{8} C^{\alpha\beta} C^{\gamma\delta} (D_\alpha D_\gamma F) \cdot (D_\beta D_\delta G),
\end{aligned} \tag{2.9}$$

where  $|F| = 1$  if  $F$  is odd (fermionic) and  $|F| = 0$  if  $F$  is even (bosonic). The second line is in fact the definition of the multiplication  $\mu_\star$ . No higher powers of  $C^{\alpha\beta}$  appear since derivatives  $D_\alpha$  are Grassmanian. The  $\star$ -product (2.9) is associative<sup>2</sup>, noncommutative and in the zeroth order in the deformation parameter  $C_{\alpha\beta}$  it reduces to the usual pointwise multiplication. One should also note that it is not hermitian,

$$(F \star G)^* \neq G^* \star F^*. \tag{2.11}$$

The  $\star$ -product (2.9) leads to

$$\begin{aligned}
\{\theta^\alpha \star \theta^\beta\} &= C^{\alpha\beta}, \quad \{\bar{\theta}_\alpha \star \bar{\theta}_\beta\} = \{\theta^\alpha \star \bar{\theta}_\alpha\} = 0, \\
[x^m \star x^n] &= -C^{\alpha\beta} (\sigma^{mn} \varepsilon)_{\alpha\beta} \bar{\theta} \bar{\theta}, \\
[x^m \star \theta^\alpha] &= -i C^{\alpha\beta} \sigma_{\beta\dot{\beta}}^m \bar{\theta}^{\dot{\beta}}, \quad [x^m \star \bar{\theta}_\alpha] = 0.
\end{aligned} \tag{2.12}$$

The deformed superspace is generated by the usual bosonic and fermionic coordinates (2.2) while the deformation is contained in the new product (2.9). From (2.12) it follows that both fermionic and bosonic part of the superspace are deformed. This is different from [8] where only fermionic part of the superspace was deformed.

The deformed infinitesimal SUSY transformation is defined in the following way

$$\delta_\xi^\star F = (\xi Q + \bar{\xi} \bar{Q}) F. \tag{2.13}$$

Since the coproduct is not deformed, the usual Leibniz rule follows. The  $\star$ -product of two superfields is again a superfield; its transformation law is given by

$$\begin{aligned}
\delta_\xi^\star (F \star G) &= (\xi Q + \bar{\xi} \bar{Q}) (F \star G) \\
&= (\delta_\xi^\star F) \star G + F \star (\delta_\xi^\star G).
\end{aligned} \tag{2.14}$$

Being interested in a deformation of the Wess-Zumino model, we need to analyze properties of the  $\star$ -products of chiral fields. A chiral field  $\Phi$  fulfills  $\bar{D}_{\dot{\alpha}} \Phi = 0$ , where

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<sup>2</sup>The associativity of the  $\star$ -product follows from the cocycle condition [14] which the twist  $\mathcal{F}$  has to fulfill

$$\mathcal{F}_{12}(\Delta \otimes id) \mathcal{F} = \mathcal{F}_{23}(id \otimes \Delta) \mathcal{F}, \tag{2.10}$$

where  $\mathcal{F}_{12} = \mathcal{F} \otimes 1$  and  $\mathcal{F}_{23} = 1 \otimes \mathcal{F}$ . It can be shown that the twist (2.6) indeed fulfills this condition, see [15] for details.

$\bar{D}_{\dot{\alpha}} = -\bar{\partial}_{\dot{\alpha}} - i\theta^{\alpha}\sigma_{\alpha\dot{\alpha}}^m\partial_m$  and  $\bar{D}_{\dot{\alpha}}$  is related to  $D_{\alpha}$  by the usual complex conjugation. In terms of the component fields,  $\Phi$  is given by

$$\begin{aligned}\Phi(x, \theta, \bar{\theta}) &= A(x) + \sqrt{2}\theta^{\alpha}\psi_{\alpha}(x) + \theta\theta H(x) + i\theta\sigma^l\bar{\theta}(\partial_l A(x)) \\ &\quad - \frac{i}{\sqrt{2}}\theta\theta(\partial_m\psi^{\alpha}(x))\sigma_{\alpha\dot{\alpha}}^m\bar{\theta}^{\dot{\alpha}} + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}(\square A(x)).\end{aligned}\quad (2.15)$$

The  $\star$ -product of two chiral fields reads

$$\begin{aligned}\Phi \star \Phi &= \Phi \cdot \Phi - \frac{1}{8}C^{\alpha\beta}C^{\gamma\delta}D_{\alpha}D_{\gamma}\Phi D_{\beta}D_{\delta}\Phi \\ &= \Phi \cdot \Phi - \frac{1}{32}C^2(D^2\Phi)(D^2\Phi) \\ &= A^2 - \frac{C^2}{2}H^2 + 2\sqrt{2}A\theta^{\alpha}\psi_{\alpha} \\ &\quad - i\sqrt{2}C^2H\bar{\theta}_{\dot{\alpha}}\bar{\sigma}^{m\dot{\alpha}\alpha}(\partial_m\psi_{\alpha}) + \theta\theta(2AH - \psi\psi) \\ &\quad + C^2\bar{\theta}\bar{\theta}\left(-H\square A + \frac{1}{2}(\partial_m\psi)\sigma^m\bar{\sigma}^l(\partial_l\psi)\right) \\ &\quad + i\theta\sigma^m\bar{\theta}\left(\partial_m(A^2) + C^2H\partial_m H\right) \\ &\quad + i\sqrt{2}\theta\theta\bar{\theta}_{\dot{\alpha}}\bar{\sigma}^{m\dot{\alpha}\alpha}(\partial_m(\psi_{\alpha}A)) \\ &\quad + \frac{\sqrt{2}}{2}\bar{\theta}\bar{\theta}C^2(-H\theta\square\psi + \theta\sigma^m\bar{\sigma}^n\partial_n\psi\partial_m H) \\ &\quad + \frac{1}{4}\theta\theta\bar{\theta}\bar{\theta}(\square A^2 - \frac{1}{2}C^2\square H^2),\end{aligned}\quad (2.16)$$

where  $C^2 = C^{\alpha\beta}C^{\gamma\delta}\varepsilon_{\alpha\gamma}\varepsilon_{\beta\delta}$ . Due to the  $\bar{\theta}$ ,  $\bar{\theta}\bar{\theta}$  and the  $\theta\bar{\theta}\bar{\theta}$  terms in (2.16),  $\Phi \star \Phi$  is not a chiral field. Following the method developed in [8] we decompose all  $\star$ -products of the chiral fields into their irreducible components by using the projectors defined in [16].

Finally, the deformed Wess-Zumino action is constructed by requiring that the action is invariant under the deformed SUSY transformations (2.13) and that in the commutative limit it reduces to the undeformed Wess-Zumino action. In addition, we try to make a minimal deformation in the sense that we deform by  $\star$ -multiplication only the terms already present in the undeformed Wess-Zumino action. However, as we shall see later, renormalizability will in fact imply the addition of some 'nonminimal' terms. Thus, we propose the following action

$$\begin{aligned}S &= \int d^4x \left\{ \Phi^+ \star \Phi \Big|_{\theta\theta\bar{\theta}\bar{\theta}} + \left[ \frac{m}{2} \left( P_2(\Phi \star \Phi) \Big|_{\theta\theta} + 2a_1 P_1(\Phi \star \Phi) \Big|_{\bar{\theta}\bar{\theta}} \right) \right. \right. \\ &\quad + \frac{\lambda}{3} \left( P_2(P_2(\Phi \star \Phi) \star \Phi) \Big|_{\theta\theta} + 3a_2 P_1(P_2(\Phi \star \Phi) \star \Phi) \Big|_{\bar{\theta}\bar{\theta}} \right. \\ &\quad + 2a_3 (P_1(\Phi \star \Phi) \star \Phi) \Big|_{\theta\theta\bar{\theta}\bar{\theta}} + 3a_4 P_1(\Phi \star \Phi) \star \Phi^+ \Big|_{\bar{\theta}\bar{\theta}} \\ &\quad \left. \left. + 3a_5 \bar{C}^2 P_2(\Phi \star \Phi) \star \Phi^+ \Big|_{\theta\theta\bar{\theta}\bar{\theta}} \right) + \text{c.c.} \right\}.\end{aligned}\quad (2.17)$$



Coefficients  $a_1, \dots, a_5$  are real and constant. Compared with the action constructed in [7], action (2.17) has two additional terms: the terms with coefficients  $a_4$  and  $a_5$ . Both terms are SUSY invariant and vanish in the commutative limit. Note that the vanishing of the  $a_5$ -term in the commutative limit was done by hand by multiplication with  $\bar{C}^2$ . These terms were not considered in [7] because their presence was not required by renormalizability of the two-point function. In addition, they are a deformation of a term not present in the commutative Wess-Zumino action. However, we shall see in the following sections that on the level of three-point functions, the  $a_3$ -term generates divergences of the form  $P_1(\Phi \star \Phi) \star \Phi^+ \Big|_{\bar{\theta}\bar{\theta}}$  while the  $a_1$ -term and the  $a_4$ -term generate divergences of the form  $P_2(\Phi \star \Phi) \star \Phi^+ \Big|_{\theta\theta\bar{\theta}\bar{\theta}}$ . In order to absorb these divergences one needs to introduce the  $a_4$ -term and the  $a_5$ -term in the action (2.17) from the very beginning.

### 3 The one-loop effective action and the supergraph technique

In this section we look at the quantum properties of our model. To be more precise, we calculate the one-loop divergent part of the effective action up to second order in the deformation parameter. We use the background field method, dimensional regularization and the supergraph technique. Note that the use of the supergraph technique significantly simplifies calculations.

In order to apply the supergraph technique, the classical action (2.17) has to be rewritten as an integral over the whole superspace. The kinetic part takes the form

$$S_0 = \int d^8z \left\{ \Phi^+ \Phi + \left[ -\frac{m}{8} \Phi \frac{D^2}{\square} \Phi + \frac{ma_1 C^2}{8} (D^2 \Phi) \Phi + c.c. \right] \right\}, \quad (3.18)$$

while the interaction is given by

$$\begin{aligned} S_{int} = \lambda \int d^8z \left\{ -\Phi^2 \frac{D^2}{12\square} \Phi + \frac{a_2 C^2}{8} \Phi \Phi (D^2 \Phi) \right. \\ \left. - \frac{a_3 C^2}{48} (D^2 \Phi) (D^2 \Phi) \Phi + \frac{a_4 C^2}{8} \Phi (D^2 \Phi) \Phi^+ \right. \\ \left. + a_5 \bar{C}^2 \Phi \Phi \Phi^+ + c.c. \right\}, \end{aligned} \quad (3.19)$$

with  $f(x) \frac{1}{\square} g(x) = f(x) \int d^4y G(x-y) g(y)$ . Following the idea of the background field method, we split the chiral and antichiral superfields into their classical and quantum parts

$$\Phi \rightarrow \Phi + \Phi_q, \quad \Phi^+ \rightarrow \Phi^+ + \Phi_q^+ \quad (3.20)$$

and integrate over the quantum superfields in the path integral.

Since  $\Phi_q$  and  $\Phi_q^+$  are chiral and antichiral fields, they are constrained by

$$\bar{D}_{\dot{\alpha}}\Phi_q = D_{\alpha}\Phi_q^+ = 0.$$

One can introduce unconstrained superfields  $\Sigma$  and  $\Sigma^+$  such that

$$\begin{aligned}\Phi_q &= -\frac{1}{4}\bar{D}^2\Sigma, \\ \Phi_q^+ &= -\frac{1}{4}D^2\Sigma^+.\end{aligned}\tag{3.21}$$

Note that we do not express the background superfields  $\Phi$  and  $\Phi^+$  in terms of  $\Sigma$  and  $\Sigma^+$ , only the quantum parts  $\Phi_q$  and  $\Phi_q^+$ . After integrating over the quantum superfields, the result will be expressed in terms of the (anti)chiral superfields. This is a big advantage of the background field method (and the supergraph technique). From (3.21) we see that the unconstrained superfields are determined up to a gauge transformation

$$\begin{aligned}\Sigma &\rightarrow \Sigma + \bar{D}_{\dot{\alpha}}\bar{\Lambda}^{\dot{\alpha}}, \\ \Sigma^+ &\rightarrow \Sigma^+ + D^{\alpha}\Lambda_{\alpha},\end{aligned}\tag{3.22}$$

with the gauge parameter  $\Lambda$ . In order to fix this symmetry we have to add a gauge fixing term to the action. For the gauge functions we choose

$$\begin{aligned}\chi_{\alpha} &= D_{\alpha}\Sigma, \\ \bar{\chi}_{\dot{\alpha}} &= \bar{D}_{\dot{\alpha}}\Sigma^+.\end{aligned}\tag{3.23}$$

The product  $\delta(\chi)\delta(\bar{\chi})$  in the path integral is averaged by the weight  $e^{-i\xi\int d^8z\bar{f}Mf}$ :

$$\int df d\bar{f} \delta(\chi_{\alpha} - f_{\alpha})\delta(\bar{\chi}^{\dot{\alpha}} - \bar{f}^{\dot{\alpha}})e^{-i\xi\int d^8z\bar{f}^{\dot{\alpha}}M_{\dot{\alpha}\alpha}f^{\alpha}}\tag{3.24}$$

where

$$\bar{f}^{\dot{\alpha}}M_{\dot{\alpha}\alpha}f^{\alpha} = \frac{1}{4}\bar{f}^{\dot{\alpha}}(D_{\alpha}\bar{D}_{\dot{\alpha}} + \frac{3}{4}\bar{D}_{\dot{\alpha}}D_{\alpha})f^{\alpha}\tag{3.25}$$

and the gauge fixing parameter is denoted by  $\xi$ . The gauge fixing term becomes

$$S_{gf} = -\xi\int d^8z (\bar{D}_{\dot{\alpha}}\bar{\Sigma})(\frac{3}{16}\bar{D}^{\dot{\alpha}}D^{\alpha} + \frac{1}{4}D^{\alpha}\bar{D}^{\dot{\alpha}})(D_{\alpha}\Sigma).\tag{3.26}$$

It is easy to show that the ghost fields are decoupled.

The part of the classical gauge fixed action quadratic in quantum superfields is

$$S^{(2)} = \frac{1}{2}\int d^8z \begin{pmatrix} \Sigma & \Sigma^+ \end{pmatrix} (\mathcal{M} + \mathcal{V}) \begin{pmatrix} \Sigma \\ \Sigma^+ \end{pmatrix}\tag{3.27}$$

where the kinetic and the interaction terms are collected in the matrices  $\mathcal{M}$  and  $\mathcal{V}$  respectively. Matrix  $\mathcal{M}$  is given by

$$\mathcal{M} = \begin{pmatrix} -m\Box^{1/2}(1 - a_1C^2\Box)P_- & \Box(P_2 + \xi(P_1 + P_T)) \\ \Box(P_1 + \xi(P_2 + P_T)) & -m\Box^{1/2}(1 - a_1\bar{C}^2\Box)P_+ \end{pmatrix}.\tag{3.28}$$

Matrix  $\mathcal{V}$  has the form

$$\mathcal{V} = \begin{pmatrix} F & G \\ \bar{G} & \bar{F} \end{pmatrix}, \quad (3.29)$$

with matrix elements

$$\begin{aligned} F &= -\frac{\lambda}{2}\Phi\bar{D}^2 + \frac{\lambda a_2 C^2}{2}\Phi\Box\bar{D}^2 + \frac{\lambda a_2 C^2}{4}(\Box\Phi)\bar{D}^2 \\ &\quad - \frac{\lambda a_3 C^2}{128}\bar{D}^2(D^2\Phi)D^2\bar{D}^2 + \frac{\lambda a_4 C^2}{64}\bar{D}^2(\Phi^+)D^2\bar{D}^2 \\ &\quad + \frac{\lambda a_5}{8}\bar{C}^2(\bar{D}^2\Phi^+)\bar{D}^2, \\ G &= \frac{\lambda a_4}{64}\bar{D}^2\left[C^2(D^2\Phi) + \bar{C}^2(\bar{D}^2\Phi^+)\right]D^2 \\ &\quad + \frac{\lambda a_5}{8}\bar{D}^2\left[C^2\Phi^+ + \bar{C}^2\Phi\right]D^2, \\ \bar{F} &= -\frac{\lambda}{2}\Phi^+D^2 + \frac{\lambda a_2 \bar{C}^2}{2}\Phi^+\Box D^2 + \frac{\lambda a_2 \bar{C}^2}{4}(\Box\Phi^+)D^2 \\ &\quad - \frac{\lambda a_3 \bar{C}^2}{128}\bar{D}^2(\bar{D}^2\Phi^+)\bar{D}^2D^2 + \frac{\lambda a_4 \bar{C}^2}{64}\bar{D}^2(\Phi)\bar{D}^2D^2 \\ &\quad + \frac{\lambda a_5}{8}C^2(D^2\Phi)D^2, \\ \bar{G} &= \frac{\lambda a_4}{64}\bar{D}^2\left[\bar{C}^2(\bar{D}^2\Phi^+) + C^2(D^2\Phi)\right]\bar{D}^2 \\ &\quad + \frac{\lambda a_5}{8}\bar{D}^2\left[\bar{C}^2\Phi + C^2\Phi^+\right]\bar{D}^2. \end{aligned} \quad (3.30)$$

The one-loop effective action is

$$\Gamma = S_0 + S_{int} + \frac{i}{2}\text{Tr} \log(1 + \mathcal{M}^{-1}\mathcal{V}). \quad (3.31)$$

The last term in (3.31) is the one-loop correction to the effective action. In order to calculate it we have to invert  $\mathcal{M}$  [16],

$$\mathcal{M}^{-1} = \begin{pmatrix} A & B \\ \bar{B} & \bar{A} \end{pmatrix} = \begin{pmatrix} \frac{m(1-a_1\bar{C}^2\Box)D^2}{4\Box f(\Box)} & \frac{D^2\bar{D}^2}{16\Box f(\Box)} + \frac{\bar{D}^2D^2-2\bar{D}D^2\bar{D}}{16\xi\Box^2} \\ \frac{\bar{D}^2D^2}{16\Box f(\Box)} + \frac{D^2\bar{D}^2-2D\bar{D}^2D}{16\xi\Box^2} & \frac{m(1-a_1C^2\Box)\bar{D}^2}{4\Box f(\Box)} \end{pmatrix} \quad (3.32)$$

where

$$f(\Box) = \Box - m^2 + m^2 a_1 (C^2 + \bar{C}^2)\Box. \quad (3.33)$$

Expansion of the logarithm in (3.31) gives the one-loop correction to the effective action

$$\Gamma_1 = \frac{i}{2}\text{Tr} \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} (\mathcal{M}^{-1}\mathcal{V})^n = \sum_{n=1}^{\infty} \Gamma_1^{(n)}. \quad (3.34)$$

## 4 Two-point and three-point Green functions

Let us now look at the divergent parts of the effective action. The first term in the expansion (3.34) is given by

$$\Gamma_1^{(1)} = \frac{i}{2} \text{Tr}(\mathcal{M}^{-1} \mathcal{V}) = \frac{i}{2} \text{Tr} \left[ AF + \bar{A} \bar{F} + B \bar{G} + \bar{B} G \right] = 0. \quad (4.35)$$

This means that the tadpole contribution to the one-loop effective action vanishes just as in the undeformed Wess-Zumino model.

The second term in (3.34) contains two classical fields and gives the one-loop divergent part of the two-point function. It is given by

$$\begin{aligned} \Gamma_1^{(2)} &= -\frac{i}{4} \text{Tr}(\mathcal{M}^{-1} \mathcal{V})^2 \\ &= -\frac{i}{4} \text{Tr} \left[ AF AF + \bar{A} \bar{F} \bar{A} \bar{F} + 2AF B \bar{G} + 2\bar{A} \bar{F} \bar{B} G \right. \\ &\quad \left. + 2AG \bar{B} F + 2\bar{A} \bar{G} B \bar{F} + 2B \bar{F} \bar{B} F \right]. \end{aligned} \quad (4.36)$$

In our previous paper [7] we obtained the result for the divergent part of the two-point function applying the background field method to the action written in terms of the component fields and assuming that  $C^2 = \bar{C}^2$ . Here we will not make this assumption. Terms in the classical action (3.18)-(3.19) which are proportional to  $a_2$  give divergences which do not have their counterparts in the classical action and would lead to a nonrenormalizable two-point function. Therefore, we set  $a_2 = 0$ . Note that this is in agreement with [7].

The divergent parts of terms appearing in (4.36) are given by

$$\text{Tr}(AF AF) \Big|_{dp} = \frac{im^2 \lambda^2 C^2}{4\pi^2 \varepsilon} \int d^8 z \Phi(z) \left[ a_3 D^2 \Phi(z) - 2a_4 \Phi^+(z) \right], \quad (4.37)$$

$$\text{Tr}(\bar{A} \bar{F} \bar{A} \bar{F}) \Big|_{dp} = \frac{im^2 \lambda^2 \bar{C}^2}{4\pi^2 \varepsilon} \int d^8 z \Phi^+(z) \left[ a_3 \bar{D}^2 \Phi^+(z) - 2a_4 \Phi(z) \right], \quad (4.38)$$

$$\begin{aligned} \text{Tr}(AF B \bar{G}) \Big|_{dp} &= \text{Tr}(AG \bar{B} F) \Big|_{dp} \\ &= -\frac{im \lambda^2 C^2 a_4}{16\pi^2 \varepsilon} \int d^8 z \Phi(z) D^2 \Phi(z) \\ &\quad - \frac{im \lambda^2 C^2 a_5}{2\pi^2 \varepsilon} \int d^8 z \Phi^+(z) \Phi(z), \end{aligned} \quad (4.39)$$

$$\begin{aligned} \text{Tr}(\bar{A} \bar{F} \bar{B} G) \Big|_{dp} &= \text{Tr}(\bar{A} \bar{G} B \bar{F}) \Big|_{dp} \\ &= -\frac{im \lambda^2 \bar{C}^2 a_4}{16\pi^2 \varepsilon} \int d^8 z \Phi^+(z) \bar{D}^2 \Phi^+(z) \\ &\quad - \frac{im \lambda^2 \bar{C}^2 a_5}{2\pi^2 \varepsilon} \int d^8 z \Phi^+(z) \Phi(z), \end{aligned} \quad (4.40)$$

$$\begin{aligned} \left. \text{Tr}(B\bar{F}\bar{B}F) \right|_{dp} &= \frac{i\lambda^2(1 - 2a_1m^2(C^2 + \bar{C}^2))}{2\pi^2\varepsilon} \int d^8z \Phi^+(z)\Phi(z) \\ &\quad - \frac{i\lambda^2a_5}{8\pi^2\varepsilon} \int d^8z \left[ C^2\Phi(D^2\Phi) + \bar{C}^2\Phi^+(\bar{D}^2\Phi^+) \right]. \end{aligned} \quad (4.41)$$

From (4.37-4.41) and (4.36) we obtain

$$\begin{aligned} \left. \Gamma_1^{(2)} \right|_{dp} &= \frac{\lambda^2(1 - (2a_1 + \frac{a_4}{2} + \frac{2a_5}{m})m^2(C^2 + \bar{C}^2))}{4\pi^2\varepsilon} \int d^8z \Phi^+(z)\Phi(z) \\ &\quad + \frac{\lambda^2(m^2a_3 - ma_4 - a_5)}{16\pi^2\varepsilon} \int d^8z \left[ C^2\Phi(z)D^2\Phi(z) + c.c. \right]. \end{aligned} \quad (4.42)$$

This result can be easily rewritten in terms of the component fields. Choosing  $a_4 = a_5 = 0$  one sees that (4.42) is in agreement with the results obtained in [7].

Let us next consider the divergent part of the three-point Green function

$$\begin{aligned} \Gamma_1^{(3)} &= \frac{i}{6} \text{Tr}(\mathcal{M}^{-1}\mathcal{V})^3 \\ &= \frac{i}{6} \text{Tr} \left[ AF AF AF + \bar{A}\bar{F}\bar{A}\bar{F}\bar{A}\bar{F} + 3AF AFB\bar{G} + 3\bar{A}\bar{F}\bar{A}\bar{F}\bar{B}G + 3AFAG\bar{B}\bar{F} \right. \\ &\quad + 3\bar{A}\bar{F}\bar{A}\bar{G}\bar{B}\bar{F} + 3AFB\bar{F}\bar{B}F + 3\bar{A}\bar{F}\bar{B}F\bar{B}\bar{F} + 3AFB\bar{F}\bar{A}\bar{G} \\ &\quad \left. + 3\bar{A}\bar{F}\bar{B}FAG + 3B\bar{G}\bar{B}\bar{F}\bar{B}F + 3\bar{B}G\bar{B}F\bar{B}\bar{F} \right]. \end{aligned} \quad (4.43)$$

The traces appearing in (4.43) are equal to

$$\begin{aligned} \left. \text{Tr}(AFB\bar{F}\bar{B}F) \right|_{dp} &= \frac{im\lambda^3a_1\bar{C}^2}{\pi^2\varepsilon} \int d^8z \Phi(z)\Phi(z)\Phi^+(z) \\ &\quad - \frac{im\lambda^3a_3C^2}{2\pi^2\varepsilon} \int d^8z \Phi(z)\Phi^+(z)D^2\Phi(z) \\ &\quad + \frac{im\lambda^3a_4C^2}{\pi^2\varepsilon} \int d^8z \Phi(z)\Phi^+(z)\Phi^+(z), \end{aligned} \quad (4.44)$$

$$\begin{aligned} \left. \text{Tr}(\bar{A}\bar{F}\bar{B}F\bar{B}\bar{F}) \right|_{dp} &= \frac{im\lambda^3a_1C^2}{\pi^2\varepsilon} \int d^8z \Phi^+(z)\Phi^+(z)\Phi(z) \\ &\quad - \frac{im\lambda^3a_3\bar{C}^2}{2\pi^2\varepsilon} \int d^8z \Phi^+(z)\Phi(z)\bar{D}^2\Phi^+(z) \\ &\quad + \frac{im\lambda^3a_4\bar{C}^2}{\pi^2\varepsilon} \int d^8z \Phi^+(z)\Phi(z)\Phi(z), \end{aligned} \quad (4.45)$$

$$\begin{aligned} \left. \text{Tr}(B\bar{G}\bar{B}\bar{F}\bar{B}F) \right|_{dp} &= \left. \text{Tr}(\bar{B}G\bar{B}F\bar{B}\bar{F}) \right|_{dp} \\ &= \frac{i\lambda^3a_4}{8\pi^2\varepsilon} \int d^8z \left[ C^2\Phi(z)\Phi^+(z)D^2\Phi(z) + c.c. \right] \\ &\quad + \frac{im\lambda^3a_5}{\pi^2\varepsilon} \int d^8z \left[ \bar{C}^2\Phi^+(z)\Phi(z)\Phi(z) + c.c. \right]. \end{aligned} \quad (4.46)$$

All other terms in (4.43) are convergent. We obtain

$$\begin{aligned} \Gamma_1^{(3)} \Big|_{dp} &= -\frac{\lambda^3(ma_1 + ma_4 + 2a_5)}{2\pi^2\varepsilon} \int d^8z [\bar{C}^2\Phi(z)\Phi(z)\Phi^+(z) + c.c.] \\ &\quad + \frac{\lambda^3(2ma_3 - a_4)}{8\pi^2\varepsilon} \int d^8z [C^2\Phi(z)\Phi^+(z)D^2\Phi(z) + c.c.]. \end{aligned} \quad (4.47)$$

From (4.47) we see that the  $a_3$ -term generates divergences of the type

$$\int d^8z [C^2\Phi(z)\Phi^+(z)D^2\Phi(z) + c.c.]$$

while the  $a_1$ -term and the  $a_4$ -term generate divergence of the type

$$\int d^8z [\bar{C}^2\Phi(z)\Phi(z)\Phi^+(z) + c.c.].$$

These divergences cannot be canceled unless we introduce two additional terms in the action (2.17).

## 5 Four-point Green function

Before making the final statement about renormalizability we still have to check whether divergences in the four-point function can be canceled. The four-point function is given by

$$\begin{aligned} \Gamma_1^{(4)} &= -\frac{i}{8}\text{Tr}(\mathcal{M}^{-1}\mathcal{V})^4 \\ &= -\frac{i}{8}\text{Tr}\left[AFAFAF + \bar{A}\bar{F}\bar{A}\bar{F}\bar{A}\bar{F} + 2B\bar{F}\bar{B}FB\bar{F}\bar{B}F\right] \\ &\quad -\frac{i}{2}\text{Tr}\left[AFAFB\bar{F}\bar{B}F + \bar{A}\bar{F}\bar{A}\bar{F}\bar{B}FB\bar{F} + AFB\bar{F}\bar{A}\bar{F}\bar{B}F + AFAFAFB\bar{G} \right. \\ &\quad + \bar{A}\bar{F}\bar{A}\bar{F}\bar{A}\bar{F}\bar{B}G + AFAFB\bar{F}\bar{A}\bar{G} + \bar{A}\bar{F}\bar{A}\bar{F}\bar{B}FAG + AFAFAG\bar{B}F \\ &\quad + \bar{A}\bar{F}\bar{A}\bar{F}\bar{A}\bar{G}B\bar{F} + AFB\bar{G}B\bar{F}\bar{B}F + \bar{A}\bar{F}\bar{B}G\bar{B}FB\bar{F} + AFB\bar{F}\bar{B}FB\bar{G} \\ &\quad + \bar{A}\bar{F}\bar{B}FB\bar{F}\bar{B}G + AFB\bar{F}\bar{A}\bar{F}\bar{A}\bar{G} + \bar{A}\bar{F}\bar{B}FAFAG + AFB\bar{F}\bar{B}G\bar{B}F \\ &\quad \left. + \bar{A}\bar{F}\bar{B}FB\bar{G}B\bar{F} + B\bar{F}\bar{B}FB\bar{F}\bar{A}\bar{G} + \bar{B}FB\bar{F}\bar{B}FAG\right]. \end{aligned} \quad (5.48)$$

There is only one non-vanishing divergent term in (5.48) and it is given by

$$\begin{aligned} \text{Tr}(B\bar{F}\bar{B}FB\bar{F}\bar{B}F) \Big|_{dp} &= \frac{i\lambda^4}{\pi^2\varepsilon} \int d^8z \left[ \bar{C}^2\Phi(z)\Phi(z)\Phi^+(z) \left( a_3\bar{D}^2\Phi^+(z) \right. \right. \\ &\quad \left. \left. - 2a_4\Phi(z) \right) + c.c. \right]. \end{aligned} \quad (5.49)$$

Therefore the divergent part of the four-point function does not vanish and it is given by

$$\Gamma_1^{(4)} \Big|_{dp} = \frac{\lambda^4}{8\pi^2\varepsilon} \int d^8z [\bar{C}^2\Phi(z)\Phi(z)\Phi^+(z) (a_3\bar{D}^2\Phi^+(z) - 2a_4\Phi(z)) + c.c.]. \quad (5.50)$$

We see that this term does not appear in the classical action (2.17). One can check that the five-point function is convergent.

Let us note that in the undeformed Wess-Zumino model divergences appear only in the two-point function. They lead to renormalization of the superfield and there is no mass counterterm. Three-point and higher-point functions are convergent, which means that there are no divergent counterterms for the coupling constants; all redefinitions can be expressed in terms of the field strength renormalization  $Z$ . We see that introducing the deformation (2.6) changes this behavior: we obtain divergences both in the three-point and in the four-point functions.

## 6 Discussion

From (4.42), (4.47) and (5.50) we see that the two-point and the three-point functions are renormalizable, while the four-point function is not. Thus, the model with arbitrary coefficients  $a_1, \dots, a_5$  is not renormalizable. The situation is very similar to the one in the noncommutative gauge theory with the massless Dirac fermion  $\psi$  [18], where only two-point Green functions were renormalizable. In that model the four- $\psi$  divergence appears and it cannot be removed.

However, there is a special choice of the coefficients  $a_1, \dots, a_5$  which renders the model renormalizable. If we fix  $a_3 = a_4 = 0$ , the divergent part of the four-point function vanishes. In that case the divergent parts of the two- and three-point functions are

$$\begin{aligned} \Gamma_1^{(2)} \Big|_{dp} &= \frac{\lambda^2(1 - (2a_1 + \frac{2a_5}{m})m^2(C^2 + \bar{C}^2))}{4\pi^2\epsilon} \int d^8z \Phi^+(z)\Phi(z) \\ &\quad - \frac{\lambda^2 a_5}{16\pi^2\epsilon} \int d^8z [C^2\Phi(z)D^2\Phi(z) + c.c.] , \end{aligned} \quad (6.51)$$

$$\Gamma_1^{(3)} \Big|_{dp} = -\frac{\lambda^3(ma_1 + 2a_5)}{2\pi^2\epsilon} \int d^8z [\bar{C}^2\Phi(z)\Phi(z)\Phi^+(z) + c.c.] . \quad (6.52)$$

All divergences in (6.51) and (6.52) have the same form as terms in the classical action (3.18)-(3.19). But this is only a necessary condition for a theory to be renormalizable; we still have to check the consistency of the field and the coupling constants redefinitions. As usual, we add counterterms to the classical action (3.18)-(3.19). The bare action is given by

$$S_B = S_0 + S_{int} - \Gamma_1^{(2)} \Big|_{dp} - \Gamma_1^{(3)} \Big|_{dp} . \quad (6.53)$$

The two-point Green function in (6.53) gives the renormalization of the superfield  $\Phi$

$$\Phi_0 = \sqrt{Z}\Phi , \quad (6.54)$$

where

$$Z = 1 - \frac{\lambda^2}{4\pi^2\epsilon} \left[ 1 - 2m(C^2 + \bar{C}^2)(ma_1 + a_5) \right] \quad (6.55)$$

and

$$m = Zm_0 \quad (6.56)$$

since  $\delta_m = 0$ . In addition to the field redefinition we obtain the redefinition of the coupling constant

$$a_{10}C_0^2 = a_1C^2 \left[ 1 + \frac{\lambda^2 a_5}{2\pi^2 \epsilon a_1 m} \right]. \quad (6.57)$$

From the three-point Green function in (6.53) we obtain the following conditions

$$\lambda = Z^{3/2} \lambda_0, \quad (6.58)$$

$$a_{50}\bar{C}_0^2 = a_5\bar{C}^2 \left[ 1 + \frac{\lambda^2 (ma_1 + 2a_5)}{2\pi^2 \epsilon a_5} \right]. \quad (6.59)$$

Taking these results into account we see that indeed for  $a_3 = a_4 = 0$  our model is renormalizable.

Let us note that for the special cases  $a_5 = -\frac{1}{2}ma_1$  and  $a_5 = 2ma_1$  equations (6.57) and (6.59) can be reduced to

$$a_{10}C_0^2 = a_1C^2 \left[ 1 - \frac{\lambda^2}{4\pi^2 \epsilon} \right]. \quad (6.60)$$

The case  $a_5 = -\frac{1}{2}ma_1$  is more interesting. With this choice the divergent part of the three-point function (6.52) vanishes. This means that there are no divergent counterterms for the coupling constants, i.e. all redefinitions are expressed in terms of the field strength renormalization

$$\begin{aligned} m &= Zm_0, \\ \lambda &= Z^{3/2} \lambda_0, \\ a_{10}C_0^2 &= a_1C^2 Z. \end{aligned} \quad (6.61)$$

These results resemble those which are valid for the undeformed Wess-Zumino model.

## 7 Conclusions

In order to see how a deformation by twist affects renormalizability of the Wess-Zumino model, we consider a special example of the twist, (2.6). Compared with the classical SUSY Hopf algebra, the twisted SUSY Hopf algebra is unchanged. In particular, the Leibniz rule (2.14) remains undeformed. The notion of chirality is however lost and we have to apply the method of projectors introduced in [8] to obtain the action. A deformation of the commutative Wess-Zumino action which is SUSY invariant and has a good commutative limit is introduced and its renormalizability properties are discussed.

Using the background field method and the supergraph technique we calculate the divergent parts of the two-point, three-point and four-point functions up to second order in the deformation parameter  $C_{\alpha\beta}$ . For one-point and two-point functions the



obtained results are in agreement with the results of [7], where the analysis was done in component fields. There is no tadpole diagram, no mass renormalization and all fields are renormalized in the same way. These results are the same as the results valid for the undeformed Wess-Zumino model. However, unlike in the undeformed Wess-Zumino model, divergences in the three-point and four-point functions appear. The divergences appearing in the three-point functions have their counterparts in the classical action while the counterparts for the divergences of the four-point function do not exist in the classical action. The five-point function is convergent.

Our results show that in general case with arbitrary coefficients  $a_1, \dots, a_5$  the model is not renormalizable. However, there is a special choice  $a_3 = a_4 = 0$  which renders renormalizability. With this choice we still have a nontrivial deformation due to the  $a_1$ -term and the  $a_5$ -term in the action. The divergent part of the four-point function vanishes and the two-point and three-point functions are renormalizable. In the special case  $a_5 = -\frac{1}{2}ma_1$  the divergent part of the three-point function also vanishes. Equations (6.57) and (6.59) are consistent with this choice and results (6.61) resemble the results from the undeformed Wess-Zumino model.

Having in mind results of [19], we also investigated on-shell renormalizability of the general model. In general, on-shell renormalizability leads to a one-loop renormalizable  $S$ -matrix. On the other hand, one-loop renormalizable Green functions spoil renormalizability at higher loops. After using the equations of motion to obtain on-shell divergent terms in our model, we are left with some non-vanishing five-point divergences proportional to  $a_3$  and  $a_4$ . Since the five-point Green function is convergent, these five-point divergences cannot be canceled unless  $a_3 = a_4 = 0$ . But we already know that in that case the model is fully renormalizable.

Renormalization of the deformed Wess-Zumino model has been previously studied in the literature, see for example [20], [21]. The models considered there are non-hermitian and have half of the supersymmetry of the corresponding  $\mathcal{N} = 1$  theory. In general they are not power-counting renormalizable, however it has been argued in [20], [21] that they are nevertheless renormalizable since only a finite number of additional terms needs to be added to the action to absorb divergences to all orders. This fact is related to the non-hermiticity of the relevant actions. Renormalizability of the deformed Wess-Zumino model in [20] was achieved by adding a finite number of additional terms to the original classical action and using the equations of motion to eliminate auxiliary field  $F$  from the theory.

The models [20], [21] are different from the model we study here. The main difference lies in the deformation, resulting in a different  $\star$ -product and a different deformed action. The action of our model is hermitian<sup>3</sup> and moreover, invariant under the full  $\mathcal{N} = 1$  SUSY. Since our model is more complicated than those discussed in [20] it is not obvious which terms should be added in the general case with arbitrary  $a_3$  and  $a_4$  in order to cancel all one-loop divergences, and whether the number of the added terms is finite. This remains to be investigated in future.

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<sup>3</sup>The deformation (2.9) is not hermitian. However, adding the complex conjugate terms by hand we obtain the hermitian action (3.18)-(3.19).

Another problem which we plan to address is the choice of deformation. Namely, renormalizability can be chosen as a criterion to test the deformation. We could chose a different deformation compared to that discussed in this paper. Using our principles (SUSY invariance, commutative limit, minimal deformation) we could construct an invariant action and check whether the obtained model has a better behavior. This could give us an important insight into which deformation of the superspace is preferred.

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## A Calculation of traces

In this appendix we give calculations of some of the traces appearing in  $\Gamma_1^{(3)}$  and  $\Gamma_1^{(4)}$ . Up to second order in the deformation parameter the trace  $\text{Tr}(AFB\bar{F}\bar{B}F)$  is given by

$$\begin{aligned}
& \text{Tr}(AFB\bar{F}\bar{B}F) = \\
& = \int \prod_{i=1}^4 d^8 z_i \left( \frac{m(1 - a_1 \bar{C}^2 \square) D^2}{4 \square (\square - m^2)} \right)_1 \delta(z_1 - z_2) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_2 \\
& \quad \cdot \left( \frac{D^2 \bar{D}^2}{16 \square (\square - m^2)} \right)_2 \delta(z_2 - z_3) \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_3 \\
& \quad \cdot \left( \frac{\bar{D}^2 D^2}{16 \square (\square - m^2)} \right)_3 \delta(z_3 - z_4) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_4 \delta(z_4 - z_1) \\
& + \int \prod_{i=1}^4 d^8 z_i \left( \frac{m D^2}{4 \square (\square - m^2)} \right)_1 \delta(z_1 - z_2) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_2 \\
& \quad \cdot \left( \frac{D^2 \bar{D}^2}{16 \square (\square - m^2)} \right)_2 \delta(z_2 - z_3) \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_3 \\
& \quad \cdot \left( \frac{\bar{D}^2 D^2}{16 \square (\square - m^2)} \right)_3 \delta(z_3 - z_4) \\
& \quad \cdot \left[ \bar{D}^2 \left( -\frac{\lambda C^2}{128} (a_3 D^2 \Phi - 2a_4 \Phi^+) \right) D^2 \bar{D}^2 \right]_4 \delta(z_4 - z_1) \\
& + \int \prod_{i=1}^4 d^8 z_i \left( \frac{m D^2}{4 \square (\square - m^2)} \right)_1 \delta(z_1 - z_2) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_2 \\
& \quad \cdot \left( \frac{D^2 \bar{D}^2}{16 \square (\square - m^2)} \right)_2 \delta(z_2 - z_3) \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_3 \\
& \quad \cdot \left( \frac{\bar{D}^2 D^2}{16 \square (\square - m^2)} \right)_3 \delta(z_3 - z_4) \left( \frac{\lambda a_5 \bar{C}^2}{8} (\bar{D}^2 \Phi^+) \bar{D}^2 \right)_4 \delta(z_4 - z_1) \\
& + \int \prod_{i=1}^4 d^8 z_i \left( \frac{m D^2}{4 \square (\square - m^2)} \right)_1 \delta(z_1 - z_2) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_2 \\
& \quad \cdot \left( \frac{D^2 \bar{D}^2}{16 \square (\square - m^2)} \right)_2 \delta(z_2 - z_3) \left[ \bar{D}^2 \left( -\frac{\lambda \bar{C}^2}{128} (a_3 \bar{D}^2 \Phi^+ - 2a_4 \Phi) \right) \bar{D}^2 D^2 \right]_3 \\
& \quad \cdot \left( \frac{\bar{D}^2 D^2}{16 \square (\square - m^2)} \right)_3 \delta(z_3 - z_4) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_4 \delta(z_4 - z_1) \\
& + \int \prod_{i=1}^4 d^8 z_i \left( \frac{m D^2}{4 \square (\square - m^2)} \right)_1 \delta(z_1 - z_2) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_2 \\
& \quad \cdot \left( \frac{D^2 \bar{D}^2}{16 \square (\square - m^2)} \right)_2 \delta(z_2 - z_3) \left( \frac{\lambda a_5 C^2}{8} (D^2 \Phi) D^2 \right)_3 \\
& \quad \cdot \left( \frac{\bar{D}^2 D^2}{16 \square (\square - m^2)} \right)_3 \delta(z_3 - z_4) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_4 \delta(z_4 - z_1)
\end{aligned}$$

$$\begin{aligned}
& + \int \prod_{i=1}^4 d^8 z_i \left( \frac{m D^2}{4 \square (\square - m^2)} \right)_1 \delta(z_1 - z_2) \\
& \cdot \left[ \bar{D}^2 \left( -\frac{\lambda C^2}{128} (a_3 D^2 \Phi - 2a_4 \Phi^+) \right) D^2 \bar{D}^2 \right]_2 \left( \frac{D^2 \bar{D}^2}{16 \square (\square - m^2)} \right)_2 \delta(z_2 - z_3) \\
& \cdot \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_3 \left( \frac{\bar{D}^2 D^2}{16 \square (\square - m^2)} \right)_3 \delta(z_3 - z_4) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_4 \delta(z_4 - z_1) \\
& + \int \prod_{i=1}^4 d^8 z_i \left( \frac{m D^2}{4 \square (\square - m^2)} \right)_1 \delta(z_1 - z_2) \left( \frac{\lambda a_5 \bar{C}^2}{8} (\bar{D}^2 \Phi^+) \bar{D}^2 \right)_2 \\
& \cdot \left( \frac{D^2 \bar{D}^2}{16 \square (\square - m^2)} \right)_2 \delta(z_2 - z_3) \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_3 \\
& \cdot \left( \frac{\bar{D}^2 D^2}{16 \square (\square - m^2)} \right)_3 \delta(z_3 - z_4) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_4 \delta(z_4 - z_1) .
\end{aligned}$$

After applying the  $D$ -algebra identities, we obtain

$$\begin{aligned}
& \text{Tr}(AFB\bar{F}\bar{B}F) = \\
& = \int d^4 \theta \int \prod_{i=1}^3 d^4 x_i (-8m\lambda^3) \left( \frac{1 - a_1 \bar{C}^2 \square}{\square - m^2} \right)_1 \delta(x_1 - x_2) \Phi(x_2, \theta) \\
& \cdot \left( \frac{1}{\square - m^2} \right)_2 \delta(x_2 - x_3) \Phi^+(x_3, \theta) \left( \frac{1}{\square - m^2} \right)_3 \delta(x_3 - x_1) \Phi(x_1, \theta) \\
& + \int d^4 \theta \int \prod_{i=1}^3 d^4 x_i (-2m\lambda^3 C^2) \left( \frac{1}{\square - m^2} \right)_1 \delta(x_1 - x_2) \Phi(x_2, \theta) \\
& \cdot \left( \frac{1}{\square - m^2} \right)_2 \delta(x_2 - x_3) \Phi^+(x_3, \theta) \left( \frac{\square}{\square - m^2} \right)_3 \delta(x_3 - x_1) \\
& \cdot \left( a_3 (D^2 \Phi)(x_1, \theta) - 2a_4 \Phi^+(x_1, \theta) \right) \\
& + \int d^4 \theta \int \prod_{i=1}^3 d^4 x_i (2m\lambda^3 a_5 \bar{C}^2) \left( \frac{1}{\square - m^2} \right)_1 \delta(x_1 - x_2) \Phi(x_2, \theta) \\
& \cdot \left( \frac{1}{\square - m^2} \right)_2 \delta(x_2 - x_3) \Phi^+(x_3, \theta) \left( \frac{1}{\square - m^2} \right)_3 \delta(x_3 - x_1) (\bar{D}^2 \Phi^+)(x_1, \theta) \\
& + \int d^4 \theta \int \prod_{i=1}^3 d^4 x_i (-2m\lambda^3 \bar{C}^2) \left( \frac{1}{\square - m^2} \right)_1 \delta(x_1 - x_2) \Phi(x_2, \theta) \\
& \cdot \left( \frac{1}{\square - m^2} \right)_2 \delta(x_2 - x_3) \left( a_3 (\bar{D}^2 \Phi^+)(x_3, \theta) - 2a_4 \Phi(x_3, \theta) \right) \\
& \cdot \left( \frac{\square}{\square - m^2} \right)_3 \delta(x_3 - x_1) \Phi(x_1, \theta) \\
& + \int d^4 \theta \int \prod_{i=1}^3 d^4 x_i (2m\lambda^3 a_5 C^2) \left( \frac{1}{\square - m^2} \right)_1 \delta(x_1 - x_2) \Phi(x_2, \theta)
\end{aligned}$$

$$\begin{aligned}
& \cdot \left( \frac{1}{\square - m^2} \right)_2 \delta(x_2 - x_3) (D^2 \Phi)(x_3, \theta) \left( \frac{1}{\square - m^2} \right)_3 \delta(x_3 - x_1) \Phi(x_1, \theta) \\
& + \int d^4 \theta \int \prod_{i=1}^3 d^4 x_i (-2m\lambda^3 C^2) \left( \frac{1}{\square - m^2} \right)_1 \delta(x_1 - x_2) \\
& \cdot \left( a_3 (D^2 \Phi)(x_2, \theta) - 2a_4 \Phi^+(x_2, \theta) \right) \left( \frac{\square}{\square - m^2} \right)_2 \delta(x_2 - x_3) \\
& \cdot \Phi^+(x_3, \theta) \left( \frac{1}{\square - m^2} \right)_3 \delta(x_3 - x_1) \Phi(x_1, \theta) \\
& + \int d^4 \theta \int \prod_{i=1}^3 d^4 x_i (2m\lambda^3 a_5 \bar{C}^2) \left( \frac{1}{\square - m^2} \right)_1 \delta(x_1 - x_2) (\bar{D}^2 \Phi^+)(x_2, \theta) \\
& \cdot \left( \frac{1}{\square - m^2} \right)_2 \delta(x_2 - x_3) \Phi^+(x_3, \theta) \left( \frac{1}{\square - m^2} \right)_3 \delta(x_3 - x_1) \Phi(x_1, \theta) . \quad (1.62)
\end{aligned}$$

Transforming the previous expression to the momentum space and using the dimensional regularization we obtain (4.44).

Similarly, we find

$$\begin{aligned}
& \text{Tr}(B\bar{G}B\bar{F}\bar{B}F) = \\
& = \int \prod_{i=1}^4 d^8 z_i \left( \frac{D^2 \bar{D}^2}{16\square(\square - m^2)} \right)_1 \delta(z_1 - z_2) \left[ \overleftarrow{D}^2 \left( \frac{\lambda a_4}{64} (\bar{C}^2 \bar{D}^2 \Phi^+ + C^2 D^2 \Phi) \right) \bar{D}^2 \right]_2 \\
& \cdot \left( \frac{D^2 \bar{D}^2}{16\square(\square - m^2)} \right)_2 \delta(z_2 - z_3) \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_3 \\
& \cdot \left( \frac{\bar{D}^2 D^2}{16\square(\square - m^2)} \right)_3 \delta(z_3 - z_4) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_4 \delta(z_4 - z_1) \\
& + \int \prod_{i=1}^4 d^8 z_i \left( \frac{D^2 \bar{D}^2}{16\square(\square - m^2)} \right)_1 \delta(z_1 - z_2) \left[ \overleftarrow{D}^2 \left( \frac{\lambda a_5}{8} (\bar{C}^2 \Phi + C^2 \Phi^+) \right) \bar{D}^2 \right]_2 \\
& \cdot \left( \frac{D^2 \bar{D}^2}{16\square(\square - m^2)} \right)_2 \delta(z_2 - z_3) \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_3 \\
& \cdot \left( \frac{\bar{D}^2 D^2}{16\square(\square - m^2)} \right)_3 \delta(z_3 - z_4) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_4 \delta(z_4 - z_1) \\
& = \int d^4 \theta \int \prod_{i=1}^3 d^4 x_i (\lambda^3 a_4 \bar{C}^2) \left( \frac{\square}{\square - m^2} \right)_1 \delta(x_1 - x_2) (\bar{D}^2 \Phi^+)(x_2, \theta) \\
& \cdot \left( \frac{1}{\square - m^2} \right)_2 \delta(x_2 - x_3) \Phi^+(x_3, \theta) \left( \frac{1}{\square - m^2} \right)_3 \delta(x_3 - x_1) \Phi(x_1, \theta) \\
& + \int d^4 \theta \int \prod_{i=1}^3 d^4 x_i (\lambda^3 a_4 C^2) \left( \frac{1}{\square - m^2} \right)_1 \delta(x_1 - x_2) (D^2 \Phi)(x_2, \theta) \\
& \cdot \left( \frac{\square}{\square - m^2} \right)_2 \delta(x_2 - x_3) \Phi^+(x_3, \theta) \left( \frac{1}{\square - m^2} \right)_3 \delta(x_3 - x_1) \Phi(x_1, \theta)
\end{aligned}$$

$$\begin{aligned}
& + \int d^4\theta \int \prod_{i=1}^3 d^4x_i (8\lambda^3 a_5 \bar{C}^2) \left( \frac{\square}{\square - m^2} \right)_1 \delta(x_1 - x_2) \Phi(x_2, \theta) \\
& \quad \cdot \left( \frac{1}{\square - m^2} \right)_2 \delta(x_2 - x_3) \Phi^+(x_3, \theta) \left( \frac{1}{\square - m^2} \right)_3 \delta(x_3 - x_1) \Phi(x_1, \theta) \\
& + \int d^4\theta \int \prod_{i=1}^3 d^4x_i (8\lambda^3 a_5 C^2) \left( \frac{1}{\square - m^2} \right)_1 \delta(x_1 - x_2) \Phi^+(x_2, \theta) \\
& \quad \cdot \left( \frac{\square}{\square - m^2} \right)_2 \delta(x_2 - x_3) \Phi^+(x_3, \theta) \left( \frac{1}{\square - m^2} \right)_3 \delta(x_3 - x_1) \Phi(x_1, \theta) . \tag{1.63}
\end{aligned}$$

The divergent part of (1.63) is (4.46).

The term contributing to the divergent part of the four-point Green function is

$$\begin{aligned}
& \text{Tr}(B\bar{F}\bar{B}FB\bar{F}\bar{B}F) = \\
& = \int \prod_{i=1}^5 d^8z_i \left( \frac{D^2 \bar{D}^2}{16\square(\square - m^2)} \right)_1 \delta(z_1 - z_2) \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_2 \\
& \quad \cdot \left( \frac{\bar{D}^2 D^2}{16\square(\square - m^2)} \right)_2 \delta(z_2 - z_3) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_3 \\
& \quad \cdot \left( \frac{D^2 \bar{D}^2}{16\square(\square - m^2)} \right)_3 \delta(z_3 - z_4) \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_4 \\
& \quad \cdot \left( \frac{\bar{D}^2 D^2}{16\square(\square - m^2)} \right)_4 \delta(z_4 - z_5) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_5 \delta(z_5 - z_1) \\
& + 2 \int \prod_{i=1}^5 d^8z_i \left( \frac{D^2 \bar{D}^2}{16\square(\square - m^2)} \right)_1 \delta(z_1 - z_2) \\
& \quad \cdot \left[ \bar{D}^2 \left( -\frac{\lambda \bar{C}^2}{128} (a_3 \bar{D}^2 \Phi^+ - 2a_4 \Phi) \right) \bar{D}^2 D^2 \right]_2 \\
& \quad \cdot \left( \frac{\bar{D}^2 D^2}{16\square(\square - m^2)} \right)_2 \delta(z_2 - z_3) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_3 \left( \frac{D^2 \bar{D}^2}{16\square(\square - m^2)} \right)_3 \delta(z_3 - z_4) \\
& \quad \cdot \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_4 \left( \frac{\bar{D}^2 D^2}{16\square(\square - m^2)} \right)_4 \delta(z_4 - z_5) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_5 \delta(z_5 - z_1) \\
& + 2 \int \prod_{i=1}^5 d^8z_i \left( \frac{D^2 \bar{D}^2}{16\square(\square - m^2)} \right)_1 \delta(z_1 - z_2) \left( \frac{\lambda a_5 C^2}{8} (D^2 \Phi) D^2 \right)_2 \\
& \quad \cdot \left( \frac{\bar{D}^2 D^2}{16\square(\square - m^2)} \right)_2 \delta(z_2 - z_3) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_3 \left( \frac{D^2 \bar{D}^2}{16\square(\square - m^2)} \right)_3 \delta(z_3 - z_4) \\
& \quad \cdot \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_4 \left( \frac{\bar{D}^2 D^2}{16\square(\square - m^2)} \right)_4 \delta(z_4 - z_5) \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_5 \delta(z_5 - z_1) \\
& + 2 \int \prod_{i=1}^5 d^8z_i \left( \frac{\bar{D}^2 D^2}{16\square(\square - m^2)} \right)_1 \delta(z_1 - z_2) \\
& \quad \cdot \left[ \bar{D}^2 \left( -\frac{\lambda C^2}{128} (a_3 D^2 \Phi - 2a_4 \Phi^+) \right) D^2 \bar{D}^2 \right]_2
\end{aligned}$$

$$\begin{aligned}
& \cdot \left( \frac{D^2 \bar{D}^2}{16\Box(\Box - m^2)} \right)_2 \delta(z_2 - z_3) \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_3 \left( \frac{\bar{D}^2 D^2}{16\Box(\Box - m^2)} \right)_3 \delta(z_3 - z_4) \\
& \cdot \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_4 \left( \frac{D^2 \bar{D}^2}{16\Box(\Box - m^2)} \right)_4 \delta(z_4 - z_5) \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_5 \delta(z_5 - z_1) \\
& + 2 \int \prod_{i=1}^5 d^8 z_i \left( \frac{\bar{D}^2 D^2}{16\Box(\Box - m^2)} \right)_1 \delta(z_1 - z_2) \left( \frac{\lambda a_5 \bar{C}^2}{8} (\bar{D}^2 \Phi^+) \bar{D}^2 \right)_2 \\
& \cdot \left( \frac{D^2 \bar{D}^2}{16\Box(\Box - m^2)} \right)_2 \delta(z_2 - z_3) \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_3 \left( \frac{\bar{D}^2 D^2}{16\Box(\Box - m^2)} \right)_3 \delta(z_3 - z_4) \\
& \cdot \left( -\frac{\lambda}{2} \Phi \bar{D}^2 \right)_4 \left( \frac{D^2 \bar{D}^2}{16\Box(\Box - m^2)} \right)_4 \delta(z_4 - z_5) \left( -\frac{\lambda}{2} \Phi^+ D^2 \right)_5 \delta(z_5 - z_1) \\
& = \int \prod_{i=1}^4 d^8 z_i (16\lambda^4) \left( \frac{1}{\Box - m^2} \right)_1 e^{i(\theta_1 \sigma^n \bar{\theta}_1 + \theta_2 \sigma^n \bar{\theta}_2 - 2\theta_1 \sigma^n \bar{\theta}_2) \partial_n^1} \delta(x_1 - x_2) \\
& \cdot \Phi^+(z_2) \left( \frac{1}{\Box - m^2} \right)_2 \delta(z_2 - z_3) \Phi(z_3) \\
& \cdot \left( \frac{1}{\Box - m^2} \right)_3 e^{i(\theta_3 \sigma^n \bar{\theta}_3 + \theta_4 \sigma^n \bar{\theta}_4 - 2\theta_3 \sigma^n \bar{\theta}_4) \partial_n^3} \delta(x_3 - x_4) \Phi^+(z_4) \\
& \cdot \left( \frac{1}{\Box - m^2} \right)_4 \delta(z_4 - z_1) \Phi(z_1) \\
& + \int d^4 \theta \int \prod_{i=1}^4 d^4 x_i (8\lambda^4 \bar{C}^2) \left( \frac{\Box}{\Box - m^2} \right)_1 \delta(x_1 - x_2) \\
& \cdot (a_3 (\bar{D}^2 \Phi^+)(x_2, \theta) - 2a_4 \Phi(x_2, \theta)) \left( \frac{\Box}{\Box - m^2} \right)_2 \delta(x_2 - x_3) \Phi(x_3, \theta) \\
& \cdot \left( \frac{1}{\Box - m^2} \right)_3 \delta(x_3 - x_4) \Phi^+(x_4, \theta) \left( \frac{1}{\Box - m^2} \right)_4 \delta(x_4 - x_1) \Phi(x_1, \theta) \\
& + \int \prod_{i=1}^4 d^8 z_i (-8\lambda^4 a_5 C^2) \left( \frac{1}{\Box - m^2} \right)_1 e^{i(\theta_1 \sigma^n \bar{\theta}_1 + \theta_2 \sigma^n \bar{\theta}_2 - 2\theta_1 \sigma^n \bar{\theta}_2) \partial_n^1} \delta(x_1 - x_2) \\
& \cdot (D^2 \Phi)(z_2) \left( \frac{1}{\Box - m^2} \right)_2 \delta(z_2 - z_3) \Phi(z_3) \\
& \cdot \left( \frac{1}{\Box - m^2} \right)_3 e^{i(\theta_3 \sigma^n \bar{\theta}_3 + \theta_4 \sigma^n \bar{\theta}_4 - 2\theta_3 \sigma^n \bar{\theta}_4) \partial_n^3} \delta(x_3 - x_4) \Phi^+(z_4) \\
& \cdot \left( \frac{1}{\Box - m^2} \right)_4 \delta(z_4 - z_1) \Phi(z_1) \\
& + \int d^4 \theta \int \prod_{i=1}^4 d^4 x_i (8\lambda^4 C^2) \left( \frac{\Box}{\Box - m^2} \right)_1 \delta(x_1 - x_2) \\
& \cdot (a_3 (D^2 \Phi)(x_2, \theta) - 2a_4 \Phi^+(x_2, \theta)) \left( \frac{\Box}{\Box - m^2} \right)_2 \delta(x_2 - x_3) \Phi^+(x_3, \theta) \\
& \cdot \left( \frac{1}{\Box - m^2} \right)_3 \delta(x_3 - x_4) \Phi(x_4, \theta) \left( \frac{1}{\Box - m^2} \right)_4 \delta(x_4 - x_1) \Phi^+(x_1, \theta)
\end{aligned}$$

$$\begin{aligned}
& + \int \prod_{i=1}^4 d^8 z_i \, (-8\lambda^4 a_5 \bar{C}^2) \left( \frac{1}{\square - m^2} \right)_1 e^{-i(\theta_1 \sigma^n \bar{\theta}_1 + \theta_2 \sigma^n \bar{\theta}_2 - 2\theta_2 \sigma^n \bar{\theta}_1) \partial_n^1} \delta(x_1 - x_2) \\
& \cdot (\bar{D}^2 \Phi^+)(z_2) \left( \frac{1}{\square - m^2} \right)_2 \delta(z_2 - z_3) \Phi^+(z_3) \\
& \cdot \left( \frac{1}{\square - m^2} \right)_3 e^{-i(\theta_3 \sigma^n \bar{\theta}_3 + \theta_4 \sigma^n \bar{\theta}_4 - 2\theta_4 \sigma^n \bar{\theta}_3) \partial_n^3} \delta(x_3 - x_4) \Phi(z_4) \\
& \cdot \left( \frac{1}{\square - m^2} \right)_4 \delta(z_4 - z_1) \Phi^+(z_1) .
\end{aligned} \tag{1.64}$$

From (1.64) the divergence (5.49) follows.

All divergences appearing in (1.62), (1.63) and (1.64) are obtained using the following formulae:

$$\begin{aligned}
& \int \prod_{i=1}^3 d^4 x_i \left( \frac{\square}{\square - m^2} \right)_1 \delta(x_1 - x_2) f_1(x_2) \left( \frac{1}{\square - m^2} \right)_2 \delta(x_2 - x_3) f_2(x_3) \\
& \cdot \left( \frac{1}{\square - m^2} \right)_3 \delta(x_3 - x_1) f_3(x_1) \Big|_{dp} \\
& = \frac{i}{8\pi^2 \varepsilon} \int d^4 x \, f_1(x) f_2(x) f_3(x),
\end{aligned} \tag{1.65}$$

$$\begin{aligned}
& \int \prod_{i=1}^4 d^4 x_i \left( \frac{\square}{\square - m^2} \right)_1 \delta(x_1 - x_2) f_1(x_2) \left( \frac{\square}{\square - m^2} \right)_2 \delta(x_2 - x_3) f_2(x_3) \\
& \cdot \left( \frac{1}{\square - m^2} \right)_3 \delta(x_3 - x_4) f_3(x_4) \left( \frac{1}{\square - m^2} \right)_4 \delta(x_4 - x_1) f_4(x_1) \Big|_{dp} \\
& = \frac{i}{8\pi^2 \varepsilon} \int d^4 x \, f_1(x) f_2(x) f_3(x) f_4(x).
\end{aligned} \tag{1.66}$$

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